Comparing Models of Corporate Bankruptcy Prediction: Distance to Default vs. Z-Score

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Abstract

This paper examines the performance of two commonly applied bankruptcy prediction models, the accounting ratio-based Altman Z-Score model, and the structural Distance to Default model which currently underlies Morningstar’s Financial Health Grade for public companies (Morningstar 2008). Specifically, we tested the following:

1. The ordinal ability of each model to distinguish companies most likely to file for bankruptcy from those least likely to file for bankruptcy as measured by the Accuracy Ratio
2. The cardinal ability of each model to predict bankruptcy as measured by the bankruptcy rate of healthy-scored companies and the average rating before bankruptcy
3. The decay of the ordinal performance of each model over time as measured by the cumulative percentage change in Ordinal Score
4. The stability of the ratings of each model as measured by the Weighted Average Drift Distance

We are cognizant that the Z-Score was not intended to be used on non-manufacturing companies (Altman 2002). However, in practice we find it is commonly used to gauge the financial health of all companies. For our purposes, we found it more relevant to include non-manufacturing companies in our testing universe. We found that Distance to Default has superior ordinal and cardinal bankruptcy prediction power within our universe. It also has a more durable bankruptcy signal, but it generates less stable ratings than the Z-Score.

Introduction

Forecasting a debtor’s ability to repay its financial obligations is a crucial endeavor for lenders and investors. Answering the question, “How likely is it that my loan will be repaid on time,” is central to the valuation and asset allocation of debt portfolios. Our comparison of the Z-Score and Distance to Default models is not a contest; rather it sheds light on the strengths and weaknesses of each model while giving lenders and investors a better understanding of the tools at their disposal when evaluating creditworthiness. Particularly in the light of recent turmoil in the credit markets, it is helpful to re-evaluate the performance of these widely-known models to verify that any prior evaluations of such models still hold true.

The Z-Score, developed by Professor Edward Altman et al, is perhaps the most widely recognized and applied model for predicting financial distress (Bemmann 2005). Professor Altman developed this intuitively appealing scoring method at a time when traditional ratio analysis was losing favor with academics (Altman 1968). Using multiple discriminant analysis, Altman narrowed a list of 22 potentially significant ratios to five that, as a set, proved significant in predicting bankruptcy in his sample of 66 corporations (33 bankruptcies and 33 non-bankruptcies). Subsequent literature has criticized the Z-Score as a poorly fit model. Specifically, although each individual ratio used as predictors in the Z-Score are believed to have some bankruptcy prediction ability, the coefficients in the Z-Score calculation weaken its predictive ability to the point where it performs no better than its most predictive predictor variable (Bemmann 2005).
Since the development of the Z-Score, financial innovation has paved the way for further development of corporate bankruptcy prediction models. The option pricing model developed by Black and Scholes in 1973 and Merton in 1974 provided the foundation upon which structural credit models were built. KMV (now Moody’s KMV), was the first to commercialize the structural bankruptcy prediction model in the late 1980s. The Distance to Default is not an empirically created model, but rather a mathematical conclusion based on the assumption that a company will default on its financial obligations when its assets are worth less than its liabilities. It is also based on all of the assumptions of the Black-Scholes option pricing model, including for example, that asset returns are log-normally distributed.

There are many dimensions upon which to measure the performance of a credit scoring system, but the most comparable and relevant way to compare models with different sample sets is by measuring their ordinal ability to differentiate between companies that are most likely to go bankrupt from those that are least likely to go bankrupt (Bemmann 2005). For this reason, our primary performance indicator for both the Z-Score and Distance to Default models is the Accuracy Ratio. The Accuracy Ratio, as defined in Appendix A, is the ratio of the area between the non-predictive (random 45 degree) line and the scoring system’s curve, and the non-predictive line and the ideal scoring system’s curve in a cumulative accuracy profile. A cumulative accuracy profile plots the cumulative percentage of the bankruptcy sample that had less than or equal to a given rating prior to bankruptcy. A perfect scoring system will have an Accuracy Ratio of 1.

Our secondary performance test gauges each model’s cardinal ability to predict bankruptcy by comparing the average score before bankruptcy and the bankruptcy rate of healthy-rated companies for each model. Ideally, a credit rating model would have a very “dangerous” average rating on companies that go bankrupt and have a bankruptcy rate of zero for “safe” rated companies. In addition, we look graphically at the type I and type II errors that occur in each model.

It is also important to know how quickly the predictive power of each model’s rating decays. To test this, we have created the Ordinal Score, defined in Appendix A, which measures a company’s ordinal predictive ability over time. The slower the decay of a model’s Ordinal Score, the earlier the model will warn of potential financial distress, enhancing its usefulness to potential investors.

Finally, rating stability is key to several of the possible applications of a credit scoring system. Because many large corporate debt investors are often required to meet regulatory standards dictating the credit-quality asset allocation of their portfolios, volatile ratings would increase the transaction and portfolio monitoring costs for such investors. In most models, ordinal and cardinal accuracy are at odds with rating stability, i.e. accuracy must be sacrificed for stability and vice versa (Cantor Mann 2006). To test rating stability we created drift tables, as defined in Appendix A, from which we could calculate a weighted average drift over several time periods. The lower the weighted average drift, the more stable the model’s ratings are.

Model Descriptions

**Distance to Default**
Morningstar’s Distance to Default score is a slightly modified structural model similar to those created by Black, Scholes and Merton and commercialized by KMV – now Moody’s KMV.
Underlying the structural model is the assumption that a company’s equity can be considered an option with a strike price equal to the book value of its liabilities and a market price equal to the market value of its assets. This implies that a company is worth nothing, i.e. it has defaulted, when the market value of the assets drops below the book value of the liabilities. Based on the current market value of a company’s assets, the historical volatility of those assets, and the current book value of a company’s liabilities, one can calculate the Distance to Default using the slightly modified Morningstar methodology described in Appendix B. This model is less intuitive than the Z-Score because it does not specifically address the cash accounting values that are typically examined in a default or bankruptcy scenario. In addition, the Distance to Default model does not examine the financial covenants that would be the true determinants of whether or not distressed company defaults on its obligations.

**Z-Score**

The Z-Score model, commonly referred to as the Altman Z-Score, was developed by Professor Edward I. Altman in 1968 (Altman 2002). Although Altman et al have subsequently modified the original Z-Score model to create the Z’-Score Model, the Z”-Score Model, and the Zeta Model, the Z-Score model is still a common component of many credit rating systems, and is relevant as a benchmark for the Distance to Default model because of the wealth of research that has been performed on the Z-Score as well as the general academic and practical familiarity with the Z-Score.

The Z-Score is constructed from six basic accounting values and one market-based value. These seven values are combined into five ratios which are the pillars that comprise the Z-Score. The five pillars are combined using Equation 1 to result in a company’s Z-Score (Altman 2002).

\[
Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5
\]  

(1)

Where:

\[
X_1 = \frac{\text{Working Capital}}{\text{Total Assets}}
\]

\[
X_2 = \frac{\text{Retained Earnings}}{\text{Total Assets}}
\]

\[
X_3 = \frac{\text{Earnings Before Interest and Taxes (EBIT)}}{\text{Total Assets}}
\]

\[
X_4 = \frac{\text{Market Value of Equity}}{\text{Book Value of Total Liabilities}}
\]

\[
X_5 = \frac{\text{Sales}}{\text{Total Assets}}
\]

\[Z = \text{Overall Index or Score}\]

This formula appeals to the practitioner’s intuition because each pillar describes a different credit-relevant aspect of a company’s operations. Liquidity, cumulative profitability, asset productivity, market based financial leverage, and capital turnover are addressed by the five ratios respectively. The Z-Score presumes that each ratio is linearly related to a company’s probability of bankruptcy.
**TLTA**
As some literature has criticized the Z-Score model as an inappropriate benchmark for comparing bankruptcy prediction models because of its poor performance relative to more modern models, we decided to include a simple single-variable model (Bemmann 2005). The TLTA model is based on the ratio of Total Liabilities to Total Assets. One would expect the probability of bankruptcy to increase as this measure of capital structure leverage increases.

**Data Collection and Refinement**
The first and most important endeavor in our data collection process was to construct the largest possible list of date-company pairings for corporate bankruptcy filings. Specifically, we define the bankruptcy date as the date that the company filed for either Chapter 7 or Chapter 11 bankruptcy protection. We sourced such corporate bankruptcy data from Bloomberg’s corporate action database to arrive at a list of 502 date-company pairings corresponding to bankruptcies filed between March 1998 and June 2009. This served as our “Master Bankruptcy List.”

**Distance to Default**
The Center for Research in Security Prices (CRSP) at the University Of Chicago Booth School Of Business provided us with a set of company-date-Distance to Default records at an annual frequency dating back to 1965. The Distance to Default was calculated in accordance with Morningstar’s published methodology paper (Morningstar 2008) included in Appendix B. CRSP’s exact methodology can be viewed in Appendix C. 56 bankruptcies from the Master Bankruptcy List were within one year of a company-date-Z-Score record.

**Z-Score**
The Z-Score involves five ratios made up of seven raw data points. These ratios are shown in Equation 1. Using Morningstar’s proprietary Equity XML Output Interface (Equity XOI) database as our data source, we pulled all available fiscal-year-end ratios dating back to 1998 for the 23,069 companies listed in our database. Each company-date pairing that did not include all seven relevant data points was expunged from the data set. From the remaining data, the relevant ratios were constructed and used to calculate a Z-Score for each company-date according to Equation 1. 165 bankruptcies from the Master Bankruptcy List were within one year of a company-date-Z-Score record.

**TLTA**
The univariate TLTA model is comprised of a single unadjusted accounting ratio, Total Liabilities to Total Assets. We sourced this ratio, like the Z-Score ratios, from Morningstar’s Equity XOI database. 143 Bankruptcies from the Master Bankruptcy List were within 1-year of a company-date-TLTA record.

**Percentile Transformation**
For all of our analyses, we transformed each rating system into a set of percentiles. The percentile breakpoints for each model were calculated using all available data points spanning all available time periods. As a result, the percentile ratings in any particular year are not necessarily uniformly distributed. However, this allows direct comparison of the performance of a company in the nth decile in one year with a company in the 10th decile of another year. Across all data sets, the higher
the percentile (or decile or quintile as the results dictate), the more dangerous and less safe the company is rated (i.e. it is rated as having a higher probability of bankruptcy).

**Caveats**

Because the available data for our Altman model, Distance to Default model, and TLTA model did not all include the same company-date pairings, each data set is unique despite using the same set of bankruptcy data. As a result, the exposure to different risk factors, such as company size, company age, industry, etc. may differ from sample to sample. Consequently, this may have a destabilizing impact on the results of our models by reducing the comparability of the results. Our use of ordinal performance as our primary comparison tool should mitigate the differences in bankruptcy rates between the samples.

In addition, our Master Bankruptcy List was not comprehensive. The bankruptcies that did occur but were missing from our data set could have had a material impact on our results had they been included.

Finally, we are not using the Z-Score in the originally prescribed manner. Altman advocates that the Z-Score model should only be used for manufacturing companies (Altman 2002). Against his recommendation, our testing universe includes non-manufacturing companies, although we did exclude financials such as banks and insurance companies. This likely hurts the Z-Score’s performance relative to the other models tested.

**Model Performance Comparison**

**Ordinal Results**

The cumulative accuracy profile shown in Figure 1 provides more detail than the Accuracy Ratio alone. Specifically, we know from looking at the cumulative accuracy profile that the Z-Score holds...
its own against the Distance to Default and is superior to our simple univariate TLTA model for companies that have a low risk of bankruptcy. However, as the risk of bankruptcy increases, the Z-Score’s ordinal ranking ability deteriorates, as demonstrated by the Z-Score’s concavity between the 80th and 100th percentiles.

<table>
<thead>
<tr>
<th></th>
<th>Accuracy Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>1.00</td>
</tr>
<tr>
<td>Distance to Default</td>
<td>0.70</td>
</tr>
<tr>
<td>TL/TA</td>
<td>0.60</td>
</tr>
<tr>
<td>Z-Score</td>
<td>0.60</td>
</tr>
<tr>
<td>No Predict</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Table 1: Accuracy Ratios**

We found the Distance to Default has superior ordinal performance than the Z-Score and our simple univariate TLTA model. In addition, the Distance to Default approaches the ordinal rating accuracy of credit rating agencies Moody’s and S&P which have been estimated to have accuracy ratios from 68% to 85% and 60% to 83% respectively for large public U.S. companies (Bemmann 2005).

**Cardinal Results**

![Figure 2: Rating Distribution of Companies Filing for Bankruptcy Within 1-Year (Type I Error)](image)

Ideally, companies that go bankrupt within one year should all be in the 10th decile of a rating system, with no soon-to-be bankrupt companies rated in the 9th decile or below. Figure 2 shows the occurrence of false-negatives, or how often each model says a company is safe, when in-fact it is not. Graphically, we can see that Distance to Default outperforms TLTA, which outperforms the Z-Score.
Figure 3 is the inverse of Figure 2. It shows the occurrence of false-positives, which are clearly quite prevalent in all three models. Ideally the graph, which shows the rating distribution for companies that did not go bankrupt within one year, would have the largest percentage of companies rated in the 1st decile, and decrease as the decile increases. We do see this relationship slightly; however, the distribution of ratings is close to uniform as all deciles are within one percentage point of each other for all three models.

<table>
<thead>
<tr>
<th></th>
<th>Average Rating Before Default</th>
<th>Default Rate of Top Quintile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to Default</td>
<td>9.1</td>
<td>0.5%</td>
</tr>
<tr>
<td>Z-Score</td>
<td>8.3</td>
<td>0.6%</td>
</tr>
<tr>
<td>TLTA</td>
<td>8.2</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Table 2: Cardinal Accuracy Measures

Of the three models, Distance to Default proved to be most predictive of bankruptcy in absolute terms. On average, the most recent Distance to Default decile before a bankruptcy event was 9.1. In addition, it had the lowest occurrence of bankruptcies in its best rated quintile of companies. The Z-Score placed second in both measures, followed by TLTA.

Durability Results

As time passes, the information contained in the inputs of any particular model becomes stale. As a result, it is reasonable to expect that the ordinal ability of any bankruptcy prediction model would decay as the allowable time-horizon for bankruptcy lengthens. Figure 4 shows the ordinal score of all three models over one to ten year bankruptcy time-horizons. The ordinal score is defined in Appendix A, and can take on a value from 1 (least ordinal ability) to 10 (most ordinal ability).
Figure 4: Ordinal Score Over All Bankruptcy Time Horizons Between 1 and 10 Years

Figure 4 shows that Distance to Default has superior ordinal ability over all bankruptcy time horizons than the Z-Score or TLTA model. Over short (<2-year) and long (>5-year) time horizons the TLTA model is superior to the Z-Score model.

Figure 5: Ordinal Score Decay Over All Bankruptcy Time Horizons Between 1 and 10 Years

The cumulative change in ordinal score in Figure 5 shows us how durable of an ordinal rating each model generates. Distance to Default generates the most durable ratings because its ordinal score decays at the slowest rate of the three models. Initially the TLTA model decays at a rapid rate, but its asymptotic slope levels off over longer time periods, allowing it to outperform the Z-Score in rating durability beyond seven years.
**Drift Results**

Many users of credit scoring systems benefit from stable ratings. Regulatory or client requirements often require debt investors to maintain a certain portfolio allocation of “safe” credit instruments. For ratings to constantly flux from “safe” to “dangerous” would cause increasing portfolio management costs to such credit market investors through increased transactions and portfolio monitoring. We measured rating stability with our measure known as Weighted Average Drift Distance which is defined in Appendix A. It can take on values from 0 (least drift, most stable) to 9 (most drift, least stable).

![Weighted Average Drift Distance Over 1-, 3- and 5-Year Time Horizons](image)

**Figure 6: Weighted Average Drift Distance Over 1-, 3- and 5-Year Time Horizons**

Figure 6 shows Distance to Default is the least stable rating system, followed by the Z-Score which is followed by TLTA. This is expected, since market-based model inputs should, in most cases, be more volatile than accounting-based inputs, and Distance to Default relies the most on market-based inputs. One of the five Z-Score ratios is also market based. TLTA is purely accounting based, and as it is describes a company’s capital structure, it is probably one of the more stable accounting-based ratios available. Large changes in the TLTA ratio for most firms would probably be the result of an intentional shift in management decision-making rather than a fluctuation of business conditions. The drift tables used to construct Figure 4 are shown in Appendix D.

**Conclusion**

In light of the recent credit market turmoil, reassessing popular models of corporate bankruptcy prediction is time well-spent. We evaluated the Distance to Default and Z-Score models for their ordinal and cardinal bankruptcy prediction abilities, rating durability over time, and rating stability.

On an ordinal basis, both models maintained accuracy ratios within the range of those calculated in previous literature (Bemmann 2005). Distance to Default outperformed the Z-Score and our univariate TLTA model in both ordinal and cardinal bankruptcy prediction. Curiously, the Z-Score’s ordinal ability is nearly equal to the other two models when ranking relatively safe companies, but performs worse in situations where the probability of bankruptcy is high. All three models were
found to have significant Type I errors by classifying a large number of companies that did not go bankrupt as potentially dangerous. But Distance to Default had superior cardinal performance, as it had both a higher average rating prior to bankruptcy, and lower bankruptcy rate for “safe” rated companies than either of the other two models.

Rating durability is essential for a bankruptcy prediction model to generate actionable results. If the signal decays too rapidly to act upon, the model is useless in practice. We found that all three models produced actionable scores. However, Distance to Default generated more durable ratings as its Ordinal Score was higher over all bankruptcy time-horizons and decayed at a slower pace than either of the other two models.

Finally, rating stability is important for creditors with regulatory credit-quality requirements to meet. Distance to Default had more volatile ratings than both the Z-Score and the TLTA model. This is an intuitive result because Distance to Default relies the most on market-based inputs, and market-based inputs are usually more volatile than accounting-based inputs.

Because nearly all situations will require ordinal or cardinal accuracy before worrying about stability, we recommend the use of the Distance to Default model over the Z-Score model when trying to predict corporate bankruptcies. This of course will also depend on the availability of data for each model. Our results do not in any way condone the conclusion that all structural models outperform all empirical ratio-based models.
References


Appendix A: Definition of Terms

Cumulative Accuracy Profile (CAP): A graph of cumulative percentage of bankruptcies on the y-axis against the ordinal scoring system on the x-axis.

Accuracy Ratio: The ratio of the area between the non-predictive line and the scoring system line and the area between the non-predictive line and the ideal line in a cumulative accuracy profile graph. The higher the Accuracy Score, the better the ability of the model to distinguish companies most likely to go bankrupt from those least likely to go bankrupt.

Ordinal Score: The sum-product of the decile scores and the percentage of bankruptcies within x years corresponding to each decile. The higher the Ordinal Score, the better the ability of the model to distinguish companies most likely to go bankrupt from those least likely to go bankrupt.

\[
Ordinal \ Score_{i} = \sum_{i=1}^{10} PBK_{i,x} \times i
\]

Where:

\[
PBK_{i,x} = \frac{\# \ of \ companies \ in \ decile \ i \ that \ went \ bankrupt \ within \ x \ years}{Total \ # \ of \ companies \ that \ went \ bankrupt}
\]

This Ordinal Score can take on continuous values from 1 (least ordinal ability) to 10 (most ordinal ability).

Weighted Average Drift Distance: The sum-product of the percentage of ratings that moved from the old rating decile to a new one, and the distance of the new rating decile from the old rating decile.

\[
WADD = \sum_{i=1}^{10} \left[ \sum_{j=1}^{10} \left( PRC_{i,j} \times (j-i) \right) \right]
\]

Where:

\[
PRC_{i,j} = \frac{\# \ of \ companies \ that \ moved \ from \ decile \ i \ to \ decile \ j}{\# \ of \ companies \ that \ started \ in \ decile \ i}
\]

The Weighted Average Drift Distance can take on continuous values from 0 (no drift, maximum stability) to 10 (complete drift, no stability).

Drift Table: A table depicting the percentage of companies that started with one credit rating and ended with another for each possible starting rating. When the starting ratings are the rows of the table and the ending ratings are the columns, the rows should sum to 100%. 

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Appendix B: Morningstar’s Distance to Default Methodology

Structural models take advantage of both market information and accounting financial information. For this purpose, option pricing models based on seminal works of Black and Scholes (1973) and Merton (1974) are a natural fit. The firm’s equity can be viewed as a call option on the value of the firm’s assets. If the value of assets is not sufficient to cover the firm’s liabilities, default is expected to occur, the call option expires worthless, and the firm is turned over to its creditors.

\[
\text{Asset Value} = \text{Equity Value} + \text{Liabilities} \tag{1}
\]

The underlying premise of contingent claim models is that default occurs when the value of the firm’s assets falls below certain threshold level in relation to the firm’s liabilities. According to Merton (1974) if the firm’s liabilities consist of one zero-coupon bond with notional value \(L\), maturing in \(T\) (without any debt payment until \(T\)), and equity holders wait until \(T\) (to benefit from expected increase of the asset value), the default probability, at time \(T\), is that the value of assets is less than the value of the liabilities. To estimate this probability, the value of the firm's liability is obtained from the firm's latest balance sheet.

\[
L_t = L_s + L_l \tag{2}
\]

Where:
- \(L_t\) = Total Liability
- \(L_s\) = Short Term Liability
- \(L_l\) = Long Term Liability

Next, the probability distribution of firm's assets value at time \(T\) needs to be estimated. It is assumed that the value of firm's assets follow a log-normal distribution, i.e. the logarithm of the firm's asset value is normally distributed and the expected change in log asset values is \(\mu - \delta - \sigma^2 / 2\). The log asset value in \(T\) year therefore follows a normal distribution with following parameters:

\[
\ln A_T = N \left[ \ln A_t + \left( \mu - \delta - \frac{\sigma^2}{2} \right) (T - t), \frac{\sigma^2}{2} (T - t) \right] \tag{4}
\]

Where
- \(\mu\) is the continuously compounded expected return on assets or the asset drift and
- \(\delta\) is asset yield, expressed in terms of current assets value and is equal to
  \((\text{TTM common + preferred dividends}) / \text{Current asset value})

Next, the probability that a normally distributed variable \(x\) falls below \(z\) is given by

\[
N\{ (z - \mu) / \sigma \} \tag{5}
\]

Where \(N\) denotes the cumulative standard normal distribution.

To empirically estimate the Black Scholes probability from equation (4), we need estimates of: \(A_t, \mu\) and \(\sigma\), which are not directly observable. Though if they were known, there would be no need for using Black-Scholes, and the probability of bankruptcy (PB) and Distance to default (DD), can be expressed (McDonald 2002) as:
\[ PB = N\left\{ - \left[ \ln A_t - \ln L_t + \left( \mu - \delta - \frac{\sigma_A^2}{2} \right) (T-t) \right] / \left[ \sigma_A \sqrt{(T-t)} \right] \right\} \]

\[ = N\left\{ - \left[ \ln \left( \frac{A_t}{L_t} \right) + \left( \mu - \delta - \frac{\sigma_A^2}{2} \right) (T-t) \right] / \left[ \sigma_A \sqrt{(T-t)} \right] \right\} \]  \hspace{1cm} (5)

\[ DD = \left( \ln A_t + \left( \mu - \delta - \frac{\sigma_A^2}{2} \right) (T-t) - \ln L_t \right) / \sigma_A \sqrt{(T-t)} \]  \hspace{1cm} (6)

\[ => PB = N \left[ -DD \right] \]  \hspace{1cm} (7)

Equation (5) shows that probability of bankruptcy is a function of the distance between the value of firm's assets today and the book value of firm's total liabilities, \( \left( \frac{A_t}{L_t} \right) \) adjusted for the expected growth in asset value, asset drift, and asset yield, \( \left( \mu - \delta - \frac{\sigma_A^2}{2} \right) \) relative to asset volatility, \( \sigma_A \).

But the market value of the firm's assets is not observable and can be very different than the book value of the firm's assets. This is the \( A_t \) in equation (5). Furthermore, we do not know the volatility of the market value of the firm's assets, nor we can use the observed asset values (book values) as a proxy of the firm's market value of assets volatility, \( \sigma_A \). That's where the option pricing comes in since it implies a relationship between the unobservable \( (A_t, \sigma_A) \) and the observable variables. As long as value of the firm's assets is below the book value of the firm's liabilities, the payoff to equity holders is zero. If the value of the firm's assets is higher than book value of the firm's liabilities, equity holders receive the residual value and their payoff increases linearly as value of the firm's assets increase over time. This can be expressed as payoff of a modified (for dividends) European call option:

\[ E_T = \max \left\{ 0, A_T - L_T \right\} \]  \hspace{1cm} (8)

Assuming risk-neutrality, the equity value, \( E_t \), can be estimated by a modified (for dividends) standard Black Scholes call option formula:

\[ E_t = A_t e^{-\delta T} \cdot N \left( d_1 \right) - L_t \cdot e^{-r T} \cdot N \left( d_2 \right) + \left( 1 - e^{-\delta T} \right) A_t \]  \hspace{1cm} (9)

Where

- \( r \) is the safe rate (one year treasury yield), and
- \( N \left( d_1 \right) \) and \( N \left( d_2 \right) \) are the cumulative standard normal of \( d_1 \) and \( d_2 \).

Dividend yield, \( \delta \), added to the standard Black Scholes model in equation (9), which appears twice in the right hand side of the equation. First, term \( A_t e^{-\delta T} \) accounts for the reduction in the value of firm's assets due to dividends that are distributed at time T. Second, term \( \left( 1 - e^{-\delta T} \right) A_t \) accounts for the fact that it is the equity holders that receive the dividend – these terms do not appear in standard Black Sholes equation for valuing a call option on a dividend paying stock since dividends are not paid to option holders:

\[ d_1 = \{ \ln \left( \frac{A_t}{L_t} \right) + \left( \mu - \delta + \frac{\sigma_A^2}{2} \right) T \} / \sigma_A \sqrt{T} \]  \hspace{1cm} (10)
and

\[ d_2 = d_1 - \sigma_A \sqrt{T} = \{ \ln [A_t / L_t] + (\mu - \delta - \sigma_A^2 / 2) T / \sigma_A \sqrt{T} \} / \sigma_A \sqrt{T} \] (11)

Given the assumption of risk-neutrality the value of the call option derived from standard Black Scholes formula is not a function of firm's asset return or drift, \( \mu \). Risk-neutrality assumption in Black Scholes formula implies that assets are expected to grow at the safe rate of return and therefore only the risk free rate, \( r \), enters the Black Scholes equation. The actual probability of bankruptcy though depends on the actual distribution of future values of assets and is a function of firm's asset drift, \( \mu \) as per modified Black Scholes equation (5).

The objective is the estimation of the firm's value of asset, \( A_t \), drift, \( \mu \), and volatility, \( \sigma_A \) though we only have one equation (9) establishing a link between the two unknown values \( A_t \) and \( \sigma_A \).

There are different methods to obtain more information to estimate these two values. One approach is to come up with another equation that establishes another link between these two values. Then both equations can be simultaneously solved to determine these two values. The optimal hedge equation (12) below, which shows the equity volatility, \( \sigma_E \) is related to asset value, \( A_t \), and asset volatility, \( \sigma_A \) establishes the additional relationship between the two values. Again \( d_1 \) in equation (12) is the standard Black Scholes \( d_1 \) per equation (10). Terms \( A_t e^{-\delta T} \) in equation (12) is needed to reflect the reduction in the value of the firm's assets due to dividends that are distributed at time \( T \):

\[ \sigma_E = (A_t e^{-\delta T} N(d_1)) / E_t \] (12)

1.1.1

If we know the equity value, \( E_t \) (market price times shares outstanding), and have an estimate of equity volatility, \( \sigma_E \) (annualized standard deviation of daily stocks daily log returns), Equations (9) and (12) are two equations with two unknowns \( (A_t, \sigma_A) \) that can be solved simultaneously for a numeric solution of the firm's asset value.

Alternatively, the firm's asset value, drift and volatility can be estimated iteratively based on daily calculations of asset values and use of CAPM. By rearranging Equation (9) we obtain asset value \( A_t \):

\[ A_t = \frac{(E_t + L_t) \cdot e^{-rT} N(d_2)}{1 - e^{-\delta T} + e^{-\delta T} N(d_1)} \] (13)

These asset values can then be directly entered in the distance to default and probability of default equations (5) and (6).

The following example illustrates step by step the formulation and solution for the iterative method.

Set time horizon \( T-t = 1 \) year (Actual trailing twelve month, TTM Business days)
Set the daily equity value, \( E_t \), for the TTM by multiplying daily common stock price times shares outstanding.

Set total daily liabilities, \( L_t \), equal to latest available quarterly sum of short term liabilities, \( L_s \), and long term liabilities, \( L_l \), for the TTM – note these figures remain the same for each day and only change when a newer quarterly balance sheet becomes available during the TTM period.

Set daily gross common and preferred dividends paid in the TTM – use the record date instead of payout date and calculate TTM dividends and annual rate of daily asset yields.

Set the daily yield for one year treasuries for the TTM.

Calculate daily asset values and their volatility for the TTM – see example iteration.

Firm’s asset volatility is calculated as the annualized standard deviation of the preceding TTM (approximately 252) business daily log returns of asset values. To calculate daily log returns of asset values for the TTM period we simply take the natural log of day two asset value divided by the natural log of day one asset value and repeating the process for all 252 business days. Next we estimate asset drift, \( \mu \) using CAPM. To do this, first, asset beta is calculated as the log of slope of regression line for excess daily arithmetic returns of assets versus the market. Second the expected asset return or drift, \( \mu \) is calculated by multiplying the estimated asset beta in the previous step by the equity risk premium (assumed to be 4.8%) and add the safe rate.

2. Calculate Distance to Default and Probability of Bankruptcy

We now can directly enter the firm’s asset volatility and drift calculated from the preceding section into the distance to and probability of default Equations (5) and (6).

References


Appendix C: CRSP’s Implementation of Morningstar’s Distance to Default Methodology

A. In case of multiple share classes, DTD is calculated for the most liquid share class as defined by the largest market cap as of the rebalancing date. Then the same DTD is assigned to other share classes of the company.

B. Distance to default (DTD) is calculated for all companies included in the size-based indices with valid daily date for at least 90 trading days. If the data for 90 trading days before the date of DTD measure calculation is not available than DTD measure is set to missing.

C. Distance to default (DTD) calculation.

Timing: quarterly rebalancing date

Inputs: 252 daily values (trading days) of the following:
- company market cap
- company total liabilities (expressed from annual and quarterly data)
- dividends
- Treasury yield, annual (one-year Treasury)
- market index total return (CRSP NYSE/Alternext/NASDAQ value-weighted market index)

Calculation process:

$$ DTD = \text{NormDist.Calculate}(-dd); $$

Where

$$ \text{NormDist.Calculate}(-dd); \quad \text{the probability that an observation from the standard normal distribution is less than or equal to -dd. This is the score to be used in creating distressed/non-distressed portfolios.} $$

$$ dd = \frac{\log(\text{asset/liability}) + (\text{driftRate} - \text{dividend yield} - \text{Standard Deviation}^2 / 2)}{\text{Standard Deviation}} $$

Where

1. asset is the value of asset0 array as of rebalancing date.

Asset0 array is initialized over 252 trading days as follows:

$$ \text{asset0} = \text{MarketCap} + \text{Total Liabilities} \text{ (in comparable units)} $$

The final values of asset0 and asset1 arrays are generated as follows:

$$ \text{asset1} = \frac{(\text{MarketCap} + \text{Total Liabilities} \times e^{-\ln(1 + \text{riskfree rate})} \times \text{Normdist}(d1 - \sqrt{\text{Standard Deviation} \times 252}))}{1 + (\text{Normdist}(d1)-1) \times e^{-\ln(1 + \text{dividend yield})}} $$

Where:
• \[ d_1 = \frac{\ln(\text{asset0} / \text{Total Liabilities}) + \ln(1 + \text{risk free rate}) - \ln(1 + \text{dividend yield}) + .5 * \sqrt{\text{Standard Deviation} * 252})^2 * 1 \text{ year}}{\sqrt{\text{Standard Deviation} * 252}) * \sqrt{1 \text{ year}} \] \\

\[ m = \text{length(\text{asset0})} \]
\[ x(\text{daily_return}) = \log(\text{asset0}_{i+1}/\text{asset0}_i) \]
\[ i = 0, \ldots, n-1 \]
\[ n = m-1 = \text{length(x)} \]

\[ s_{\text{standard deviation}} = \sqrt{\frac{\sum_{i=0}^{n-1} (x_i - \bar{x})^2}{n-1}} \]

• dividend yield = sum (252 trading days of dividends * shares_outstanding) / value from asset0 array
• Standard Deviation = 252 trading days standard deviation of asset0 array
• riskfree rate = annual Treasury yield (one year Treasury)
• MarketCap = price*shares_outstanding

Asset1 values are stored in the asset1 array. A sum of squared errors is calculated between the asset0 array and asset1 array. If the sum of squared errors is not less than 0.01 then the values of the asset1 array are copied over the asset0 array. The process repeats until the sum of squared errors target is reached, or a maximum of 100 iterations is reached.

2. liability = TotalLiability

3. driftRate –

\[ \text{driftRate} \]
\[ \text{if (expectedReturn} <= 0) \text{ then driftRate} = \text{riskfreeRate} \]
\[ \text{else driftRate} = \text{Log(1 + expectedReturn)}; \]

\[ \text{if (expectedReturn} <= 0) \text{ then driftRate} = \text{riskfreeRate} \]
\[ \text{else driftRate} = \text{Log(1 + expectedReturn)}; \]

where

\[ \text{expectedReturn} = \text{riskfreeRate} + \text{beta} * 0.048 \]

\[ \text{riskfreeRate} = \text{annual Treasury yield (one year Treasury)} \]
\[ \text{beta} = \text{security market beta over 252 trading day period calculated using asset0 array values, a market index, and riskfreeRate/252 i.e. CAPM regression beta from the following:} \]
\[ \text{(daily_asset0_return - riskfreeRate/252)} = a + \text{beta(daily_market_return - riskfreeRate/252)} + e \]

driftRate is calculated as of rebalancing date after asset0 array process is completed.

D. Total Liability: The CRSP Compustat Merged Database is used to link trading securities and retrieve the company Total liability. Total liability is defined as Compustat quarterly data Item
Q54 (LTQ) or Compustat annual data Item A181 if the quarterly data is not available. The best single Compustat record (excluding LS linktypes) for the company is used for Total Liabilities value. A valid Total Liability value is defined as non-missing and non-zero (negative Total Liabilities are considered valid).

Annual and Quarterly Total Liability values are used to create a single daily record with the latter taking precedence. Up to four quarters lag is used to find non-missing value to use on a given breakpoint date.
## Appendix D: 1-Year Drift Tables

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