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# Allocation to Deferred Variable Annuities with GMWB for Life

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## Executive Summary

We study a variety of asset allocation issues associated with deferred variable annuities with guaranteed minimum withdrawal benefits for life (VA+GMWB) with the goal of developing a framework for the construction of optimal retirement portfolios. We begin by analyzing the value of the guarantee (GMWB) and its role in the portfolio context. The guarantee or rider fee will lower the net return of the VA account; however, the guarantee will provide a hedge against market downside risk, and should the investor hold the contract for the rest of their life, the VA+GMWB should provide them with an income stream for life. We present an optimization framework for making this very important tradeoff that considers the investors unique financial situation as well as their unique preferences. Over the long term horizon, the VA+GMWB return distribution is expected to be more conservative than the VA alone, which adds complexity to the determination of an optimal product mix. In the spirit of Sharpe's method for estimating the "effective mix" of a mutual fund, we develop a method for estimating the effective mix of a VA+GMWB. The effective behavior of the VA+GMWB is helpful in determining the optimal allocation to a VA+GMWB in a retirement portfolio.

The crux of the study centers on using a Monte Carlo simulation based optimization to find an optimal product-type mix of traditional investment products and a VA+GMWB by maximizing a utility function at the life expectancy. Ceteris paribus, the major findings are:

- The higher the risk tolerance, the lower the VA+GMWB allocation;
- The higher the age, the lower the VA+GMWB allocation;
- The higher the subjective life expectancy, the higher the VA+GMWB allocation;
- The higher the ratio between wealth and income gap, the lower the VA+GMWB allocation; and,
- The preference for bequest has almost no impact on the VA+GMWB allocation.

These findings provide helpful guidelines to product allocation to a VA+GMWB in a retirement portfolio.

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## Introduction

Investors are living longer than ever before; yet, there has never been more uncertainty regarding the future of what have traditionally been regarded as guaranteed sources of income, such as traditional defined-benefit pension plans and Social Security. Traditional defined-benefit pension plans and Social Security both offer the investor what is thought of as a guaranteed income stream for life during retirement. The switch from defined-benefit pensions to define-contribution plans that has taken place in corporate America switches the responsibility of providing a lifetime income stream from employers to individuals. As a result, many retirees face two related risks that threaten their financial well being in retirement:

1. Longevity risk – the risk of out-living one’s savings and facing financial ruin; and,
2. Investment performance risk – the risk that bad investment performance will reduce the value of the investor’s portfolio.

Tools for controlling these risks should be of particular interest to retirees. A large number of innovations have taken place in the development of insurance products that enable today’s retirees to help mitigate these two risks and build a more secure retirement. Unfortunately, these insurance features can be difficult to understand and from a retiree’s stand point how and when such features might be expected to assist them in retirement can be quite confusing.

The average 65-year old retiree can expect to live around another 20 years. Some retirees won’t live past 70, while record numbers are expected to live past the century mark. This creates considerable uncertainty around how much money will be needed to fund retirement. Once in retirement, a portfolio of traditional assets and the size of a prudent sustainable retirement income stream that it can support are very sensitive to market fluctuations. A series of ill timed downturns in the market could drastically increase the longevity risk faced by an investor forcing them to dramatically decrease their retirement standard of living. In the absence of cash flows, the sequence of returns is not important. In retirement, out going cash flows make the sequence of returns very important.

The fear of out-living one’s assets drives some investors to adopt unnecessarily frugal life styles. While at the other end of the spectrum, some investors will spend too much depleting their savings early into their retirement years. Most investors can avoid these extreme outcomes through a proper combination of traditional investment products, such as mutual funds or exchange traded funds (ETFs), and insurance products that offer a guaranteed income stream for life such, such as pay-out annuities or variable annuities with lifetime living benefit guarantees. A type of insurance known as longevity insurance can insure an income for life no matter how long the retiree lives.

The first widely available form of longevity insurance that helped investors control the risk of outliving their savings was pay-out annuities (immediate annuities). With pay-out annuities, an investor purchases an annuity contract with a lump sum payment and in return the insurance company promises to pay them an income stream for as long as they live. Concerns around exchanging a large amount of one’s net worth for a contract coupled with fears of an early death prevent some investors from purchasing pay-out annuities.

A new type of longevity insurance has emerged that is purchased for on going insurance fee that does not require the annuitization of the investor’s assets. Like the longevity insurance offered from payout annuities, this new form of longevity insurance, called a guaranteed minimum withdrawal benefit (GMWB) for life, can also guarantee an investor an income stream for life provided that the conditions of the longevity insurance agreement are met.

Relatively little work has been done on how best to combine traditional investments with insurance products that offer a guaranteed income stream for life to create a retirement income solution that properly mitigates the primary risks faced by retirees: longevity risk and investment performance risk. The seminal work on finding the optimal mix including immediate payout annuities was Chen and Milevsky (2003). In this article we expand and refine their model to determine the optimal split between traditional investments and variable annuities with guaranteed minimum withdrawal benefits for life (VA + GMWB).<sup>1</sup>

We believe the easiest way to incorporate this into the traditional investment process is to follow the typical strategic asset allocation decision with a secondary “product-type” optimization that determines the high level allocation to traditional products and insurance products. The optimization will simultaneously solve for A) the optimal percentage to invest in traditional products, B) the optimal percentage to invest in the VA + GMWB, and C) the detailed asset allocation model for the percentage allocated to traditional products.

We begin this analysis with an overview of the mechanics of the VA + GMWB and a procedure for estimating the value of the common features associated with the rider. In this case, the “VA” refers to a deferred variable annuity insurance contract chassis. Ignoring the potential preferred tax treatment of the deferred variable annuity legal structure, conceptually the deferred variable annuity is similar to a mutual fund.<sup>2</sup> For an additional fee, the GMWB insurance rider can be added to the base insurance contract. The GMWB alters the behavior of a portfolio making the “effective” asset allocation more conservative. We present a Monte Carlo simulation framework for estimating the “effective” asset allocation of a VA + GMWB. Finally, we present the intuition behind our guaranteed income allocator optimization framework for determining the product-type split, although the mathematical details are saved for the Appendix C.

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<sup>1</sup> We use a utility maximization framework to determine the optimal split as shown in Appendix C.

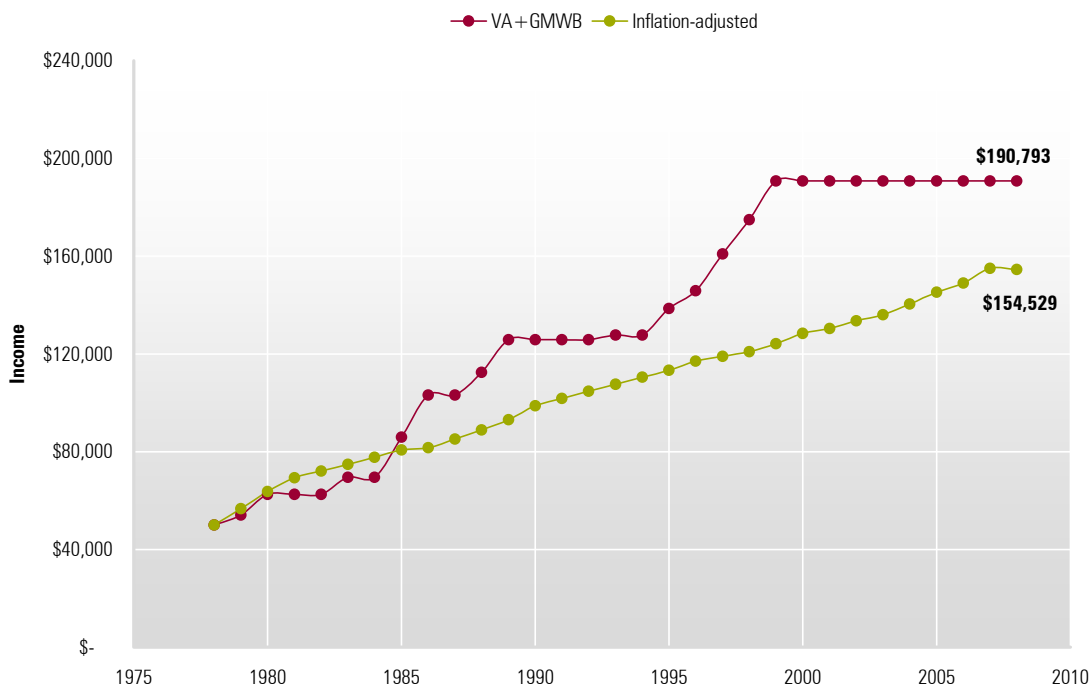
<sup>2</sup> Additional features are sometimes included as part of the base variable annuity contract and the investor can periodically switch amongst different approved investment vehicles (e.g. risk based asset allocation portfolios or combinations of different sub-accounts) without selling the contract.

## Overview of VA with Lifetime GMWB

A recent innovation in deferred variable annuity products is the guaranteed minimum withdrawal benefit (GMWB) rider. The GMWB is often referred to as a benefit rider. We will focus on the GMWB rider for life in this study, which is sometimes referred to as a Guaranteed Lifetime Withdrawal Benefit or GLWB. The GMWB rider for life gives investors the ability to protect their retirement investments against downside market risk by allowing the investor the right to withdraw a fixed percentage of the benefit base each year until death. At the time of writing, typical lifetime GMWB products offer a fixed percentage of the benefit base or guaranteed withdrawal rate of approximately 5% at age 65 even though the GMWB fee may vary for different products or companies. The guaranteed incomes continue even if the linked deferred VA account value drops to zero. The benefit base, sometime referred to as the protected base, can step up and resets to the high-water mark of the contract value on the rider anniversary date when the market has performed well. The remaining contract value at death will be paid to beneficiaries, which removes the investor concern about giving up liquidity to the heirs (i.e. if I die early, my whole family loses).

Figure 1 plots the guaranteed income assuming that one bought a hypothetical VA + GMWB contract at the end of year 1978. The inflation-adjusted equivalent is also plotted for comparison purpose. The VA asset allocation remains 80% stocks / 20% bonds for the entire period from 1979 to 2008. All fees are at the appropriate average level shown in Appendix D. The guaranteed income steps up whenever the contract value exceeds the current benefits base and never decreases, and reaches \$190,793 by 2008. The first payment of \$50,000 starts at the beginning of 1979. The stock market crash in year 2000 and the market crisis in 2008 do not reduce the income, and the guarantee keeps the income level flat. It is worth noting that over this time period, the inflation-adjusted equivalent to \$50,000 in 1979 is \$154,529 in 2008; thus, the annual income kept up with inflation.

**Figure 1. The VA + GMWB income stream from 1979 to 2008**



The corresponding contract values and benefits bases from 1979 to 2008 are shown in Figure 2. Because of strong market performance from 1979 to 1999, the benefit base steps up in a number of the years. The dynamics of the VA+GMWB contract value is documented in the Appendix A.

**Figure 2. The VA+GMWB Contract Value and Benefit Base from 1979 to 2008**

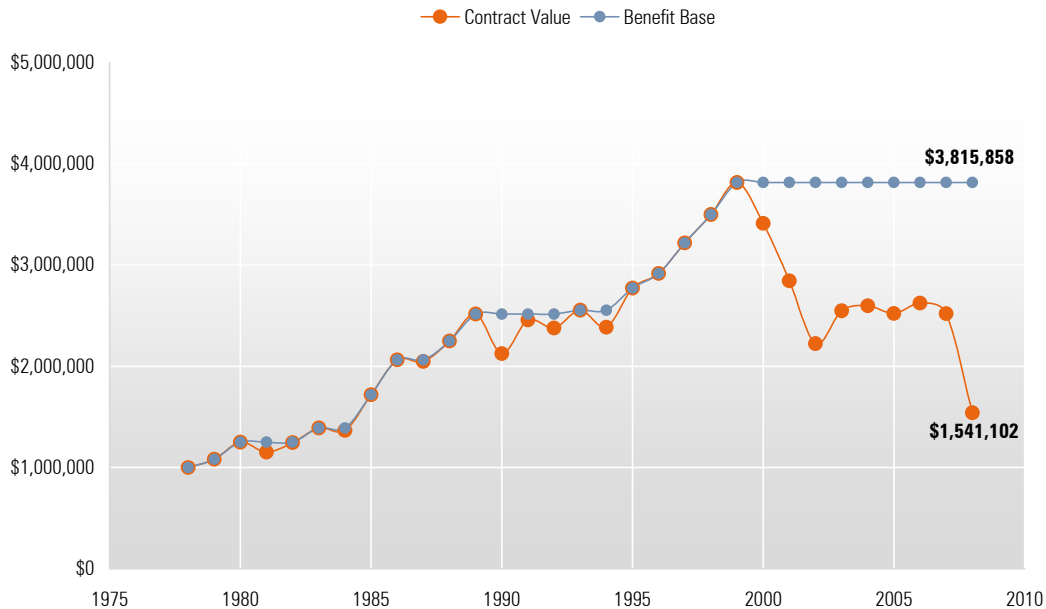


Figure 2 illustrates a couple of important points about the VA+GMWB. First, in a predominantly rising market (e.g. 1979 to 2000) the difference between the contract value and the benefit base is relatively small. Conversely, in a predominantly falling market (e.g. 2000 to 2008) the difference between the contract value and the benefit base is significant. Under such conditions, selling the contract would result in a substantial loss of future income. To realize the benefit of this type of longevity insurance, investor must approach the VA+GMWB as an investment for life.

## The Value of a Lifetime GMWB

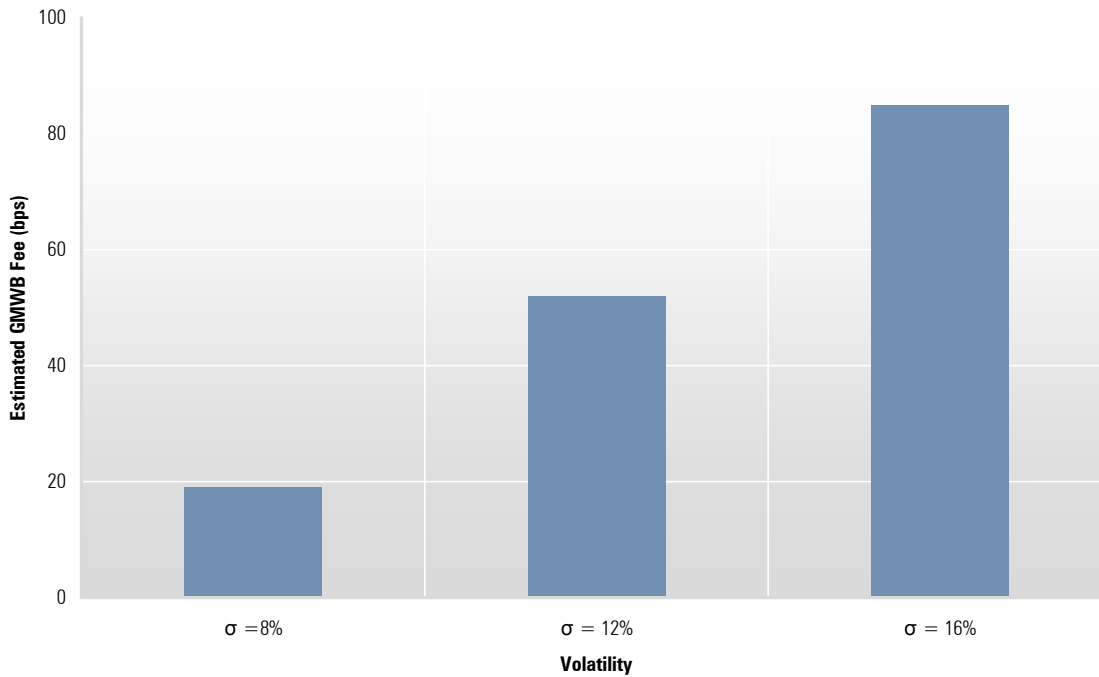
Before we can allocate assets into the VA+GMWB products, we need to understand the value of a lifetime GMWB. The value of a lifetime GMWB is directly linked to its fee. Bauer, Kling, and Russ (2008) establishes a universal pricing framework for such guaranteed benefits. Holz, Kling, and Russ (2007) applies the Bauer, Kling, and Russ (2008) framework to price the lifetime GMWB using a Monte Carlo simulation technique. In this paper, we follow Holz, Kling, and Russ (2007) to calculate the value of lifetime GMWB riders for different parameters using the deterministic withdrawal method; the details are documented in Appendix B. The deterministic withdrawal is the maximum withdrawal allowable for the lifetime GMWB contract, e.g. 5% of the benefits base. Appendix D contains the asset allocation models, capital market assumptions, and fee structures used in the simulations.

The relevant parameters that affect the value of a lifetime GMWB include the volatility of the VA account portfolio, the age of the investor or life expectancy, the maximum allowable withdrawal rate, and the frequency of the benefits base step-up.

In this paper, we only consider the deterministic withdrawal method from a less-sophisticated investor's perspective, not from an arbitrageur's perspective for two reasons. First, the surrender charge associated with the lifetime GMWB will deter investors from surrendering the rider during a surrender period. Second, Holz, Kling, and Russ (2007) shows that the value of a lifetime GMWB is only slightly higher to an arbitrageur with optimal behavior than it is to the more simplistic deterministic withdrawal behavior of a typical investor.

Figure 3 shows the relationship between the volatility of the VA asset allocation and the value of lifetime GMWB. In Figure 3, the risk-free rate is assumed to be 4% with an annual step-up feature. Standard option pricing theory states that the higher the volatility is, the higher the option price. The same conclusion holds true for the lifetime GMWB. Should insurance companies charge a fixed fee for the GMWB no matter the risk of the portfolio (the current common practice), investor's should select an aggressive asset allocation for the VA+GMWB.

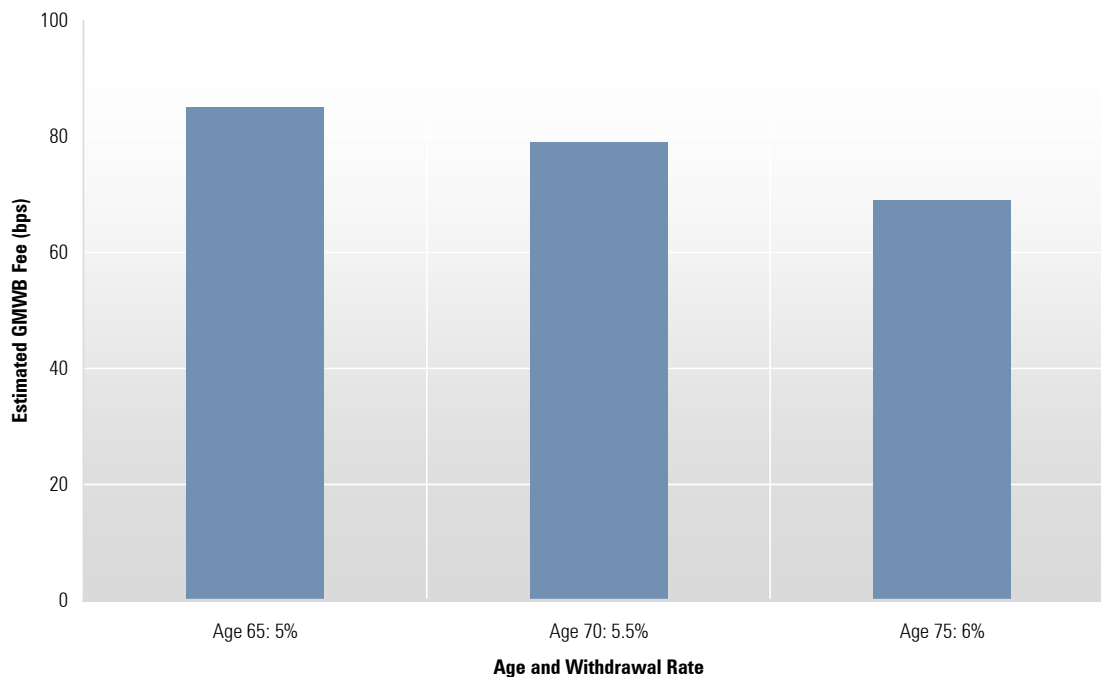
The estimated fair value of lifetime GMWB for an aggressive VA asset allocation (80% equities) is about 0.8%. Currently, the typical GMWB rider fee ranges from 0.6%-0.9% regardless of the asset allocation. Those with a conservative VA+GMWB asset allocation are probably over paying and those with an aggressive VA+GMWB asset allocation are probably getting a fairly good deal.

**Figure 3. The volatility impact on the value of the lifetime GMWB**

Holz, Kling, and Russ (2007) shows that the price for a lifetime GMWB is higher for younger investors all else the same. This result flows directly from option pricing theory; when the time (T) to expiration is long, the value of an equivalent option is higher. Holding all of the other parameters constant and assuming no step-up, the paper also shows that the price is higher with a higher maximum allowable withdrawal rate. This is intuitive as a higher income stream has a higher present value than a lower income stream.

In a typical new VA + GMWB contract, however, the maximum allowable withdrawal rate increases with the buyer's age, for example, 5% for at age 65, 5.5% at age 70, and 6% at age 75 based on our surveys. Figure 4 shows the age and maximum withdrawal rate impact on the value of lifetime GMWB from our Monte Carlo simulation results. We assume that the contract has an annual step-up feature, the VA asset allocation is 80/20, and the risk-free rate is 4%. It can be seen that even with a higher withdrawal rate of 6% at age 75, the lifetime GMWB value is still slightly less than that for the 5% withdrawal rate at age-65.<sup>3</sup> The value of a lifetime GMWB purchased at age 70 with 5.5% withdrawal rate falls between the lifetime GMWB purchased at age 65 with 5% withdrawal rate and the lifetime GMWB purchased at age 75 with 6% withdrawal rate.

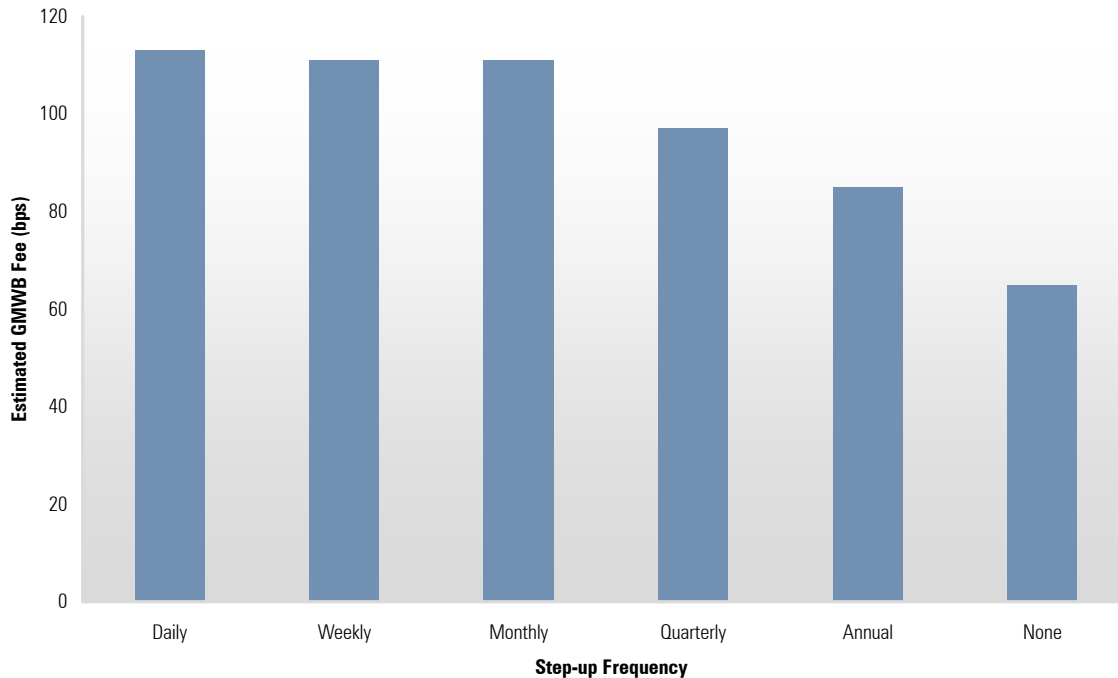
**Figure 4. The age and maximum allowable withdrawal rate impact on the value of the lifetime GMWB**



<sup>3</sup> It is easy to see how such a framework could be used to design an online "calculator" that would help investors time the purchase of the GMWB.

Figure 5 shows the step-up frequency impact on the value of lifetime GMWB. The frequency ranges from daily, weekly, monthly, to quarterly step-up (4 times a year), to annual step-up (1 time a year), and to no step-up (0 time a year). It shows that the most frequent step-up feature has the highest value, all else the same. While a quarterly step-up is often trumpeted as wonderful improvement over an annual step-up, we estimate the incremental fair value to be 12 basis points. The incremental fair value is about 15 basis points when the step-up frequency is changed from quarterly to daily. There is no significant difference among daily, weekly, and monthly step-ups.

**Figure 5. The frequency of income step-up impact on the value of the lifetime GMWB**



In summary, the longer the life expectancy is, the higher the expected value of a lifetime GMWB. The more aggressive the VA+GMWB asset allocation is, the higher the value is. The more frequent the benefits base steps-up, the higher the value. The higher the maximum withdrawal rate is, the higher the value.

Later, when we present our framework for determining the optimal product-type mix, we will focus on what we call the “all-in” fee, which includes all fees for the VA+GMWB. When shopping for a VA+GMWB investors should strive to find a VA+GMWB combination in which the stand-a-lone VA fees are comparable to the mutual fund fees and GMWB fees are close to those presented in Figures 3, 4, and 5.

## “Effective” Asset Allocation for VA+GMWB

In general, the deferred variable annuity account is invested in certain pre-defined underlying funds, and its account value rides up or down with the financial market. The list of available investment strategies may be limited should the investor seek to attach the GMWB rider to the deferred VA contract. Nevertheless, the investment strategies are subject to downside risk. The lifetime GMWB acts as a form of portfolio insurance protecting the investor from downside risk. Conceptually, this is similar to a put option that provides downside protection to a traditional portfolio. Options change the characteristics of the expected return distribution. A critical question to answer is does a lifetime GMWB enable a retiree to tolerate more equity risk for other assets?

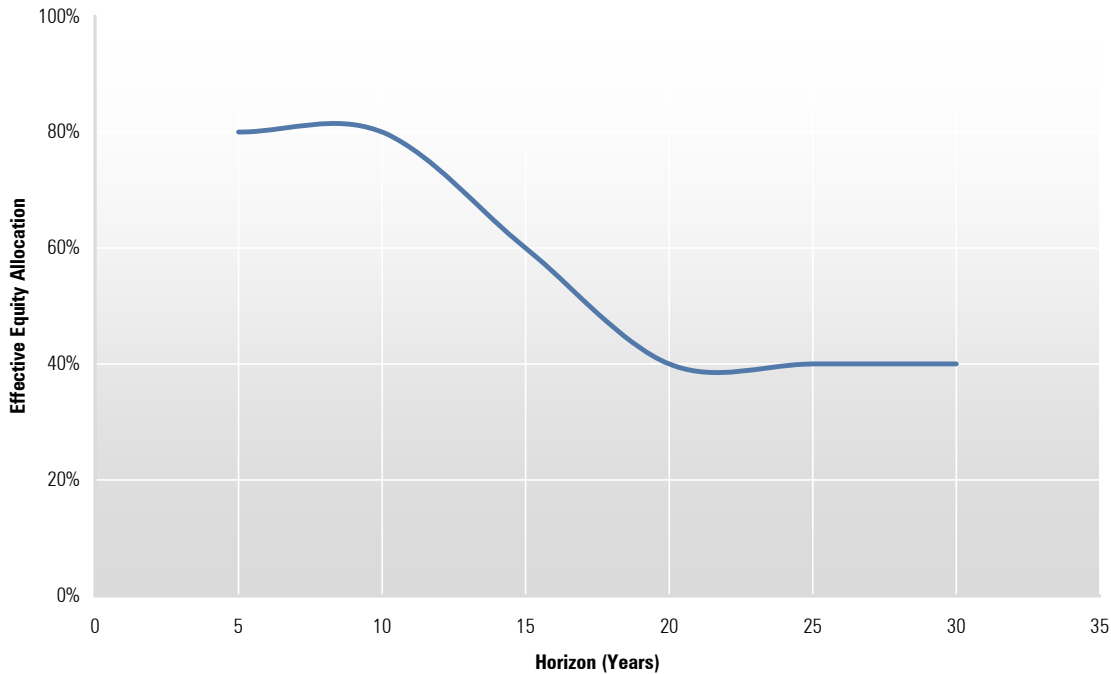
On one side, the rider fee will lower the net return of the VA account, on the other side, the rider will provide a hedge against downside risk. Therefore, over the long term horizon, the VA+GMWB return distribution is expected to be more conservative than the VA alone.

We estimate the effective asset allocation for VA+GMWB through the Monte Carlo simulations. It turns out that the effective asset allocation is dependent on the time horizon. At each period- $t$ , we calculate the value of the total benefits of the VA+GMWB at both the 50th percentile and the 5th percentile. The value of the total benefits includes the total withdrawals and the ending account value. The total withdrawals are the summation of each guaranteed income payment which is then compounded to period- $t$  at the risk-free rate. The 50<sup>th</sup> and the 5<sup>th</sup> percentiles are selected because they capture the relevant characteristics of the option feature of VA+GMWB.

We repeat the same Monte Carlo simulation procedures for a series of traditional mutual fund portfolios with different asset allocations. In each case, the income for each period is based on the income of the VA+GMWB. To determine the effective asset allocation of the VA+GMWB, we then minimize the distance of the total benefits values at both the 50th percentile and 5th percentile between the VA+GMWB and a mutual fund portfolio. The mutual fund asset allocation that minimizes this distribution-based distance is shown in Figure 6 for each year.

Figure 6 illustrates that the effective asset allocation for a VA+GMWB with an asset allocation of 80% equities and 20% bonds depends on the time horizon. The maximum allowable withdrawal rate is 5%. For a time horizon shorter than 10 years, the effective asset allocation remains unchanged at 80/20, but for a longer time horizon (> 20 years), effective asset allocation becomes more conservative at 40/60. As we will see shortly, the effective asset allocation over the life expectancy (about 20-years) is useful in determining both the optimal product allocation (VA+GMWB vs. traditional products) and the optimal asset allocation inside VA+GMWB.

**Figure 6. The effective VA+GMWB asset allocation as a function of time horizon**



One quick example should provide additional insight. Suppose that the time horizon is 30-years, the target overall strategic asset allocation is 50% equities and 50% bonds, and the investor has \$1.0 million to invest. For now, we assume that 20% of the total asset will be invested in a VA+GMWB with asset allocation of 80% equities and 20% bonds. If the impact of the embedded guarantee or option is ignored, the traditional assets will need to have an asset allocation of 43% equities and 57% bonds so that the total weighted average asset allocation is 50% stocks and 50% bonds. Now, considering that the effective asset allocation of VA+GMWB is estimated at a far more conservative 40% equities and 60% bonds at a 30-year horizon, however, the traditional assets need to have an asset allocation of 53% equities and 47% bonds to create a portfolio that behaves most like the target overall strategic asset allocation of 50% equities and 50% bonds.

## Determining the Optimum Mix

The details of our guaranteed income allocator optimization framework for determining the product-type split are presented in Appendix C. The primary factors that drive our model for determining the optimal proportion of an investor's target asset allocation that should be implemented with traditional products and variable annuities with guaranteed minimum withdrawal benefits for life are described below.

1. **Risk Tolerance** – The investor risk tolerance is an input into the model and can be determined in variety of ways. Conservative investors prefer certainty to uncertainty and therefore prefer products that offer downside protection and guaranteed income. Conversely, aggressive investors are not as concerned about downside protection and guaranteed income.
2. **Wealth vs. Total Retirement Expenses** – Investors that have large amounts of wealth relative to their total expected retirement expenses have little to no need for longevity insurance and their portfolios can withstand sustained market down turns and longevity risk without threatening their living standard.
3. **Existing Sources of Guaranteed Income vs. Annual Income Need** – Investors need to compare the amount of annual income they will receive from a traditional defined benefit pension, social security, and existing investments that offer a guaranteed income stream for life to their annual income need. If the existing sources of guaranteed income will cover a substantial part of the annual income need, there is a lower need for additional longevity risk and investment performance risk insurance.<sup>4</sup>
4. **Subjective Life Expectancy** – The longer an investor lives the greater the longevity risk and the need for longevity insurance. On average, healthy individuals with healthy life styles with relatives that tend to live longer than normal are more likely to benefit from variable annuity products with GMWB features.<sup>5</sup>
5. **Fees vs. Insurance Benefits** – The structure and fees of different insurance products vary widely. Prior to investing it is important to understand the potential insurance benefits and the costs. As similar benefits may be available in multiple products or combinations of products it is important to compare other alternatives.
6. **Likelihood of Meeting the Contract Conditions** – Investors that are likely to liquidate all or part of the contract value are less likely to benefit from the GMWB for life insurance rider. This includes investors that move from investment to investment and investors who lack the discipline to only withdraw the guaranteed amount.
7. **Desire to Leave a Bequest vs. the Desire for Lifetime Income** – For most investors, the desire to leave the maximum possible bequest to their heirs is unusual. Most investors treat a bequest as a secondary concern that ranks below the desire for a lifetime of income. Investors with unusually high bequest motives will more likely benefit from investing in traditional assets.

<sup>4</sup> "Wealth vs. Total Retirement Expenses" and "Existing Sources of Guaranteed Income vs. Annual Income Need" are measures of the individual's economic situation. We typically find it helpful to combine these two factors and focus on the ratio of Wealth divided by the annual funding gap (Annual Income Need less Existing Sources of Guaranteed Income).

<sup>5</sup> "Objective" life expectancy refers to the actuarial life expectancy. "Subjective" life expectancy is based on the individual's knowledge of their current health status, life style, and family health history.

8. **Investment Options** – For most investors this is not an issue as there are typically a large number of acceptable underlying investment options. In extraordinary circumstances, there may not be an acceptable underlying investment for a particular individual.

Most of these factors can be determined by using a specialized retirement income-oriented asset allocation questionnaire. In addition to these factors, we assume the target overall strategic asset allocation has already been determined.

These factors drive a Monte Carlo simulation-based optimization that determines the product type split that maximizes the investor's utility at the life expectancy. The optimization simultaneously solves for A) the optimal percentage to invest in traditional products, B) the optimal percentage to invest in the VA+GMWB, and C) the detailed asset allocation model for the percentage allocated to traditional products. For now we proceed with a large scale scenario test to illustrate the intuition of the model.

## Scenario Studies – A Large Scale Test

To help illustrate the impact of the different factors on the optimal product type split we programmatically ran a large number of cases or scenarios in which a large number of permutations spanning the applicable space were observed. This included 675 different permutations on different parameters such as age, risk aversion, subjective life expectancy, and preference for bequest.

In this paper, we do not limit the amount that can be allocated to the VA + GMWB. In practice, however, we put a maximum on the amount that can be allocated to the VA + GMWB to avoid what one might deem to be too much product specific risk. The main reason is that we believe there is a strong likelihood that the VA + GMWB will be implemented with a single product. There is no reason to think that such products will have an issue, but the guarantees in such cases depend on the credit worthiness of the issuer and the nature of the guarantees is that they are most likely to pay off over a relatively long time horizon during a market crisis. For example, the credit crunch of 2007 highlighted the potential danger of some money market funds – a type of investment that many investors would have deemed very safe and potential resting spots for 100% of their wealth. A basic and prudent strategy is to diversify across asset classes and across *products*. We want to avoid having a single product that could be exposed to particular unforeseen crisis dominate an individual's portfolio.

Due to the way VA + GMWBs are currently priced, we assume that the asset allocation of the VA + GMWB is fixed at 80/20 for all the scenarios and that the overall target strategic asset allocation for all assets is known.<sup>6</sup> Appendix C shows that our objective function maximizes the utility at the life expectancy (for example a 20-year horizon). We have shown that the effective asset allocation for the VA + GMWB is 40/60 for the 20-year horizon. Therefore, the following relationship holds:

$$S = P * 40\% + (1 - P) * T$$

where S is the known equity portion in the strategic asset allocation, for example, 50%. P is the unknown product allocation to VA + GMWB and T is the unknown equity allocation to the traditional asset, respectively. P and T are related to each other by the completion portfolio and both of them can be solved simultaneously by maximizing the utility function shown in Appendix C.

Overall, the following relationships are observed after the 675 cases were optimized:

- The higher the risk tolerance, the lower the VA + GMWB allocation;
- The higher the age, the lower the VA + GMWB allocation;
- The higher the subjective life expectancy, the higher the VA + GMWB allocation;
- The higher the ratio between wealth and income gap, the lower the VA + GMWB allocation; and,
- The preference for bequest has almost no impact on the VA + GMWB allocation.

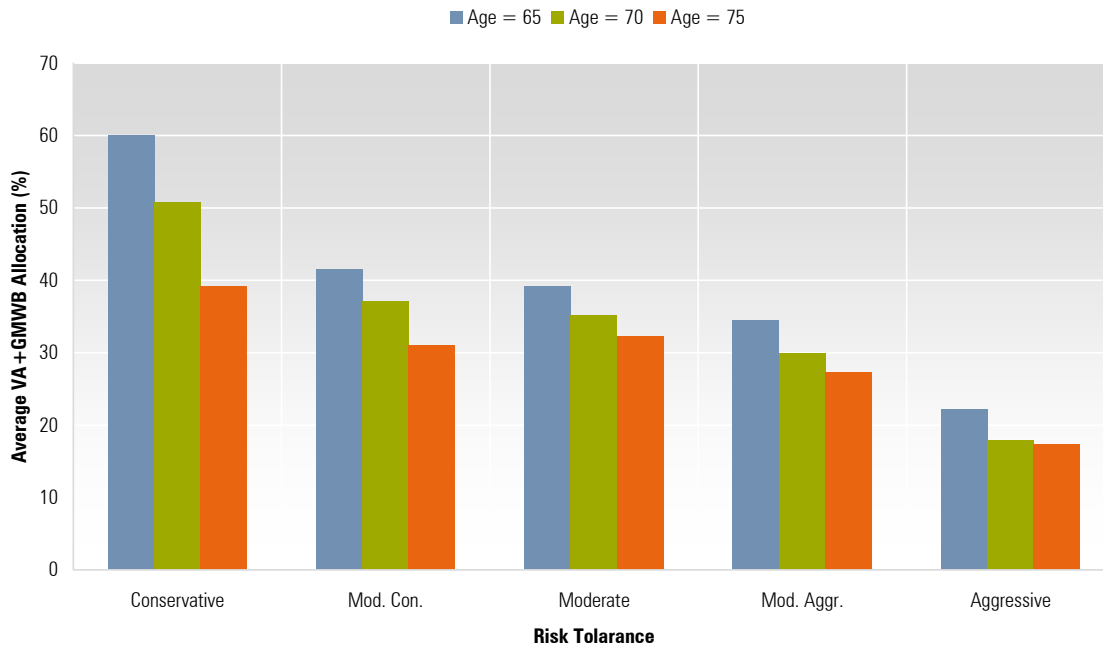
<sup>6</sup> When selecting a VA + GMWB most insurance companies allow investors to select from a series of asset allocation model portfolios that vary from conservative to moderately aggressive or from a list of sub-accounts. However, in almost all cases the insurance company limits the aggressiveness of the portfolio by allowing a maximum equity exposure of 80%.

## Risk Tolerance and Age

Figure 7 illustrates the impact of two factors: age and risk tolerance, where age is closely tied to the individual’s objective life expectancy. All else equal, conservative investors receive higher allocations to the VA + GMWB than aggressive investors and younger investors receive higher allocations to the VA + GMWB than older investors. The impact of age is consistent with the value of the GMWB on age determined earlier (see Figure 4). Again, we see that the longevity insurance from a VA + GMWB is closely related to a put option or portfolio insurance. The value of an option increases with time; thus, for younger investors there is more time for the “put option” of the VA + GMWB to be “in the money.”

Note that the withdrawal rates in Figure 7 are 5%, 5.5%, and 6% correspond to ages 65, 70, and 75, respectively, as shown in Figure 2. These withdrawal rates are current approximate industrial average rates. Higher withdrawal rate assumptions lead to higher estimated VA + GMWB values, thus higher VA + GMWB allocations, holding all other parameters constant.

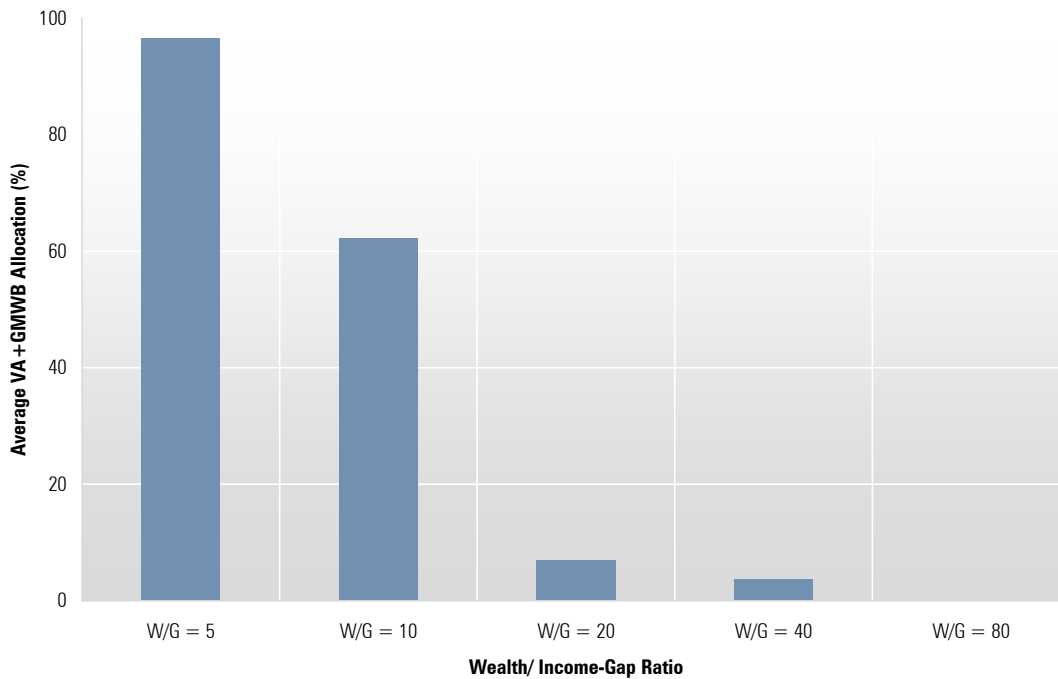
**Figure 7. The Age and Risk Tolerance Impact on the average VA + GMWB Allocation**



## Economic Situation (Wealth / (Annual Income Need less Annual Guaranteed Income))

Figure 8 illustrates the impact of the ratio of Wealth divided by the annual funding gap (Annual Income Need less Existing Sources of Guaranteed Income). This ratio captures information about the individual's "Wealth vs. Total Retirement Expenses" and "Existing Sources of Guaranteed Income vs. Annual Income Need" and serves as a collective measure of the individual's economic situation. Wealthy or well funded investors do not face longevity risk; thus, the need for longevity insurance diminishes as the investor's "funding status" improves. Intuitively, as the possibility of running out of money declines, so does the allocation to a VA+GMWB.

**Figure 8. The Ratio of Wealth to Income Gap Impact on the average VA+GMWB Allocation\***

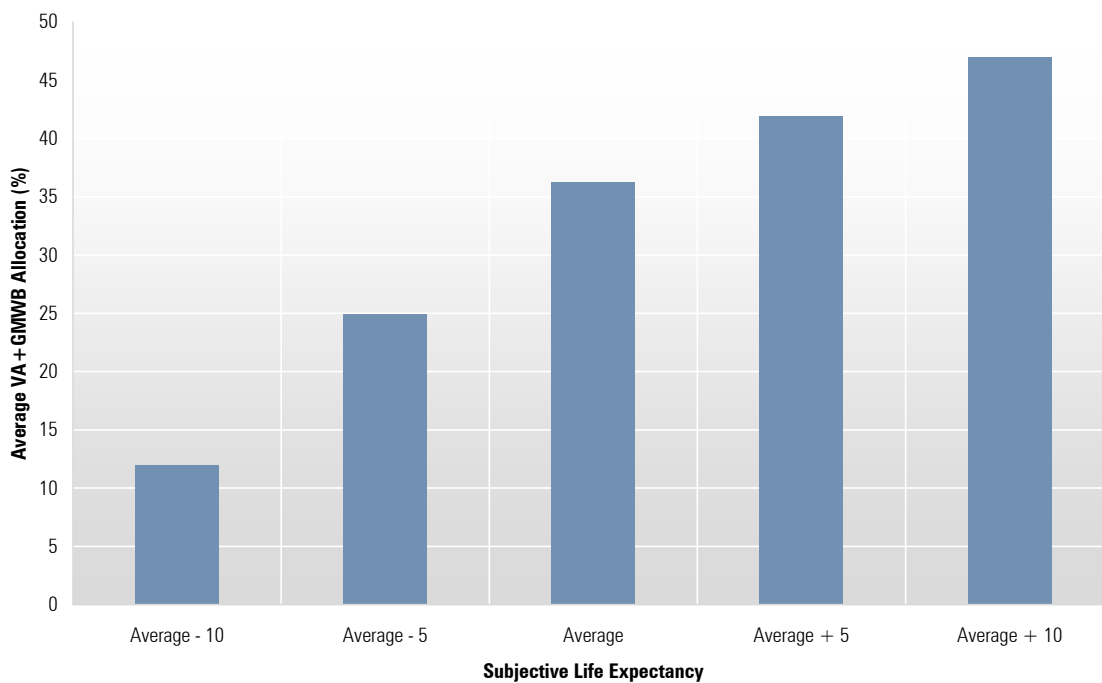


\* The average VA+GMWB allocation at W/G = 80 is 0%.

## Subjective Life Expectancy

Based on an individual's age and current mortality tables we can determine the objective life expectancy of an individual. However individuals have more detailed knowledge of their unique family history, lifestyle, and general health. These factor should enable them to make a qualitative assessment if they will live longer or shorter than average (as determined by the objective life expectancy estimate). Figure 9 illustrates the impact of the investor's assessment of their subjective life expectancy on the allocations to VA + GMWB for life. Intuitively, those that think they will live longer than the objective life expectancy face greater longevity risk and consequently need more longevity insurance; thus, all else equal they receive a larger allocation to the VA + GMWB product type.

**Figure 9 The subjective Life Expectancy Impact on the average VA + GMWB allocation \***



+/- 5 means five-year shorter / longer than average life expectancy

## Preference for Income vs. Preference for Bequest

The final factor that we analyzed during this large scale test was the impact of the individual's preference for lifetime income (personal consumption) vs. their preference to leave the largest possible bequest. In contrast with the Chen and Milevsky (2003) framework based on immediate payout annuities in which the "bequest" preference is very important, this factor has almost no impact on the amount that is allocated to the VA+GMWB product type. This is not surprising as the remaining VA+GMWB contract value is paid to the investor's beneficiary(ies) on death, which is fundamentally different from the immediate payout annuity products where this type of factor has a substantial impact on the amount allocated to insurance products.

## Conclusions

In summary, as life expectancy increases, more and more retirees will live into their late 80s and beyond. Future market performance is uncertain and exposes investors to investment performance risk. Poor market performance dramatically increases longevity risk associated with increasing life expectancy, but the degree to which retirees face longevity risk depends on a large number of factors, including: wealth, Social Security, defined benefit pensions, retirement expenses / life style, how long they live, and uncertain market performance. These factors coupled with individual preferences should be considered when determining the role of variable annuities with guaranteed minimum withdrawal benefits in a portfolio.

We have studied the characteristics of the VA + GMWB for life and built a framework to determine the optimum mix between traditional assets and VA + GMWB for a retirement portfolio. The major findings are:

- The higher the risk tolerance, the lower the VA + GMWB allocation;
- The higher the age, the lower the VA + GMWB allocation;
- The higher the subjective life expectancy, the higher the VA + GMWB allocation;
- The higher the ratio between wealth and income gap, the lower the VA + GMWB allocation; and,
- The preference for bequest has almost no impact on the VA + GMWB allocation.

Hedging longevity risk is a crucial task. Implementing a target asset allocation with a mixture of traditional investment products and variable annuities with guaranteed minimum withdrawal benefits for life creates a powerful retirement income solution that enables investors to participate in the potential upside of good markets and providing them with income for life in bad markets.

## Appendix A. Dynamics of the VA + GMWB Contract Value

While the dynamics of the traditional investments are straightforward, the dynamics of the deferred variable annuity with a guaranteed minimum withdrawal benefit for life are not. The dynamics of the VA + GMWB contract value are:

$$V_{t+1} = [V_t - fM_t - gM_t] \cdot R_t \quad (1)$$

$$R_t = e^{\mu - \varepsilon - \frac{\sigma^2}{2} + \sigma B_t}$$

$$M_t = \max(V_s, 0 \leq s \leq t)$$

$$Income_t = gM_t$$

Where  $V_t$  and  $M_t$  are the VA account or contract value and benefits base at time  $t$ , respectively.  $M_t$  is the maximum of the contract value from initial purchase to current period, i.e. the high-water mark of the contract value.  $B_t$  denotes a standard Brownian motion with mean zero and variance  $t$ .  $\sigma$  is the volatility of the VA underlying funds. The constant  $g$  is the guaranteed withdrawal rate (e.g. 5% of the benefits base). The constant  $\varepsilon$  is the total fees for the VA account including underlying fund fees and M&E&A fees, and  $\mu - \varepsilon$  is the net return of the VA account. The constant  $f$  is the GMWB rider fee (e.g. 0.8%). The spending or income is stepped up each time the contract or account value reaches a new maximum.

The  $g$  value can be age-dependent (e.g. 5% if income starts at age 65, and 5.5% if income starts at age 70.)

## Appendix B. Pricing the Lifetime GMWB

We use a Monte Carlo simulation technique to price the lifetime GMWB based on the Holz, Kling, and Russ (2007). The contract value follows equation (1) except that we need to replace  $\mu - \varepsilon$  with risk-free rate (see John C. Hull, "Options, Futures, and Other Derivatives", sixth edition, Page 294). Our goal is to find an appropriate rider fee  $f$ . The present value of total VA + GMWB value including cumulative withdrawals and ending contract value for one simulation path is:

$$PV(j, X) = \sum_{t=1}^{115-age} p_{age} \cdot q_{age+t-1} [C_t^j + D_t^j] \cdot e^{-rt} \quad (2)$$

Where

$age$ : current age; 65

115: maximum age that one can live up to

115- $age$ : investment horizon

$X$ : a withdrawal method, in this paper, it is the maximum annual withdrawal allowable for the contract

$r$ : risk-free rate; 4%

${}_{t-1} p_{age}$ : the probability for a ( $age$ )-year old to survive the next  $t$ -years

$q_{age+t-1}$ : the probability for a ( $age + t$ )-year old to die within the next year

$C_t^j$ : the future value of the cumulative withdrawals at the end of period- $t$

$D_t^j$ : the ending contract value at the end of period- $t$ .

Mortality rates are from the Society of Actuary 2000 Mortality rate table. From this table, for example, an individual at age 64 will have a 0.81 percent probability to die in the next year.

For 5000 simulations, the average present value is:

$$PV(X) = \frac{1}{5000} \sum_{j=1}^{5000} PV(j, X) \quad (3)$$

We employ binary searches so that the price to be solved for will make the average present value in equation (3) be equal to the initial purchase amount, such as \$1 million.

## Appendix C. VA + GMWB Optimization Model

We use Monte Carlo simulation-based optimization to determine the product type split that maximizes the investor's utility at the life expectancy. Again, this utility maximization objective function is similar in spirit to the objective function presented in the appendix of Chen and Milevsky (2003). The objective function to be optimized is:

$$\max_{\beta} \text{LiveState} = b + \frac{Y_T + C_T + \eta(GI_T)a_{x+T}}{I_0} \quad (4)$$

*DeathState* is the utility function:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma} \quad (5)$$

$I_0$ : the initial total wealth

*LiveState*: value at live state (a function of income)

*DeathState*: value at death state (a function of bequest)

$b$ : a constant (on the order of 1) which allows additional adjustment on preference between income and bequest. It also prevents the utility function from dividing by zero at-death values.

$A$ : the preference coefficient for being alive

$T$ : life expectancy

$P_x$ : objective probability of survival  $t$  years conditional on survival at age  $x$

$C_T$ : the future value of the total withdrawals at the life expectancy  $T$ .

$D_T$ : the ending VA+GMWB contract value at the life expectancy  $T$

$Y_T$ : the traditional portfolio balance at the life expectancy  $T$

$GI_T$ : the guaranteed income at the life expectancy  $T$

$a_{x+T}$ : the annuity factor conditional on being alive at the life expectancy  $T$

$\square$  is set at 90% to take into consideration of the liquidity premium of the guaranteed income

$\gamma$ : the coefficient of relative risk aversion, which is assessed through a series of risk questions and answers from the client.

The first term in (4) is the utility of the live state. It includes the traditional portfolio value, total cumulative withdrawals from both traditional portfolio and VA + GMWB, and the total expected future guaranteed incomes from VA + GMWB conditional on being alive at the life expectancy. The second term in (4) corresponds to the utility of the death state. It includes the ending values of both traditional portfolio and the VA + GMWB contract value. The values at both live and death states are in a relative basis because the specific utility function in (5) is most robust for relatively small values.

The objective probability of survival is adjusted based on the individual's subjective assessment of their life expectancy.

## Appendix D. Model Asset Allocations and Capital Market Assumptions for VA+GMWB

The VA+GMWB is assumed to be invested in a moderate aggressive allocation (80/20) throughout the study. The detailed asset allocations are shown in table A.

**Table A. Asset Allocations for VA+GMWB**

Asset Classes	Benchmark Proxy	Moderate Aggressive Allocation
U.S. Large Cap Stocks	Russell 1000	35%
U.S. Mid Cap Stocks	Russell MidCap	15%
U.S. Small Cap Stocks	Russell 2000	5%
International Stocks	MSCI EAFE	25%
U.S. Aggregate Bonds	LB Aggr. Bond	15%
Short-term Bonds	LB 1-5 Yr Gvt/Credit	5%
Cash	CG U.S. 3 Mo Tbill	0%

Table B shows the Ibbotson's forward-looking returns and standard deviations for the seven asset classes. Table C shows the historical correlation matrix for the seven asset classes. Table D shows the fee structure assumed in this study. For mutual fund portfolios, the total fees of 1.3% include 1.3% management fee. For the VA+GMWB, the total fees of 2.8% include 1% underlying fund management fee, 1% VA M&E&A fee, and 0.8% GMWB rider fee. The GMWB rider fee is based on the benefit base, while the other fees are based on the contract value.

**Table B. The Ibbotson's forecasted returns and forecasted standard deviations for the seven asset classes.**

Asset Classes	Ibbotson E[R]*	Ibbotson STD
U.S. Large Cap Stocks	11.03%	19.49%
U.S. Mid Cap Stocks	12.61%	22.28%
U.S. Small Cap Stocks	14.73%	28.74%
International Stocks	11.39%	24.62%
U.S. Aggregate Bonds	5.24%	7.05%
Short-term Bonds	4.10%	4.21%
Cash	3.48%	2.97%

\* Ibbotson forward-looking annual expected return (arithmetic).

**Table C. Historical correlation matrix for the seven asset classes**

	Large Cap	Mid Cap	Small Cap	International	Aggr. Bonds	S/T Bonds	Cash
Large Cap	100%	95%	83%	57%	24%	17%	1%
Mid Cap	95%	100%	93%	55%	23%	17%	-1%
Small Cap	83%	93%	100%	52%	14%	8%	-2%
International	57%	55%	52%	100%	16%	13%	-5%
Aggr. Bonds	24%	23%	14%	16%	100%	96%	11%
S/T Bonds	17%	17%	8%	13%	96%	100%	23%
Cash	1%	-1%	-2%	-5%	11%	23%	100%

**Table D. Assumed Fee Structure for Traditional Mutual Fund Portfolio and VA + GMWB**

Fees Category	Mutual Fund Portfolio	VA + GMWB
Fund Management Fee*	1.3%	1 %
VA M&E&A Fee**	N/A	1 %
GMWB Rider Fee	N/A	0.8%
Total Fees	1.3%	2.8%

\* Mutual fund has higher management fee because of additional handling and administrative expenses compared to VA.

\*\* Mortality and expense risk charges and administration fee.

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