Human Capital, Asset Allocation, and Life Insurance

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Financial planners and advisors increasingly recognize that human capital must be taken into account when building optimal portfolios for individual investors. But human capital is not simply another pre-endowed asset class; it contains a unique mortality risk in the form of the loss of future income and wages in the event of the wage earner’s death. Life insurance hedges this mortality risk, so human capital affects both optimal asset allocation and demand for life insurance. Yet, historically, asset allocation and life insurance decisions have been analyzed separately. This article develops a unified framework based on human capital that enables individual investors to make these decisions jointly.

Academics and practitioners increasingly recognize that the risk and return characteristics of human capital, such as wage and salary profiles, should be taken into account when building portfolios for individual investors. Merton (2003) pointed out the importance of including the magnitude of human capital, its volatility, and its correlation with other assets in asset allocation decisions from the perspective of personal risk management. The employees of Enron Corporation and WorldCom suffered extreme examples of this risk. Their labor income and their financial investments in the companies provided no diversification, and they were heavily affected by their companies’ collapses.

A unique aspect of an investor’s human capital is mortality risk—that is, the family’s loss of human capital in the event of the wage earner’s death. Life insurance has long been used to hedge against mortality risk. Typically, the greater the value of the human capital, the more life insurance the family demands. Intuitively, therefore, human capital affects not only optimal asset allocation but also optimal life insurance demand. These two important financial decisions are generally analyzed separately, however, in theory and practice. We found few references in either the risk/insurance literature or the investment/finance literature to the importance of considering these decisions jointly within the context of a life-cycle model of consumption and investment. In other words, popular investment and financial planning advice about how much life insurance one should carry is seldom framed in terms of the riskiness of one’s human capital. And optimal asset allocation, which has only lately started to be framed in terms of the risk characteristics of human capital, is rarely integrated with life insurance decisions.

Motivated by the need to integrate these two decisions, we merged these traditionally distinct lines of thought together in one framework. We argue that these two decisions must be determined jointly because they serve as risk substitutes when viewed from the perspective of an individual investor’s portfolio. Life insurance is a perfect hedge for human capital in the event of death because term life insurance and human capital have a negative 100 percent correlation with each other in the “alive” (consumption) state versus “dead” (bequest) state. If insurance pays off at the end of the year, human capital does not, and vice versa. Thus, the combination of the two provides great diversification to an investor’s total portfolio. Figure 1 illustrates the types of decisions the investor faces, together with the variables that affect the decisions.

The unified model we discuss is intended to provide practical guidelines for optimal asset allocation.
allocation and life insurance decisions for individual investors in their preretirement years (accumulation stage).1

Human Capital and Financial Capital
An investor’s total wealth consists of two parts. One is readily tradable financial assets; the other is human capital. Human capital is defined here as the present value of an investor’s future labor income.2 Economic theory predicts that investors make asset allocation and life insurance purchase decisions to maximize their lifetime utilities of wealth and consumption. Both of these decisions are closely linked to human capital because, although human capital is not readily tradable, it is often the single largest asset an investor has.

Typically, young investors have far more human capital than financial capital because they have more years to work than older workers and have had fewer years to save and accumulate financial wealth. Conversely, older investors tend to have more financial capital than human capital. Figure 2 illustrates the patterns of financial capital and human capital over an investor’s working (pre-retirement) years from age 25 to age 65.

Financial Asset Allocation and Human Capital. The changing mix of financial capital and human capital over a life cycle affects allocations of individuals’ financial assets. In the late 1960s, economists established models that implied that individuals should optimally maintain constant portfolio weights throughout their lives (Samuelson 1969; Merton 1969). Those models assumed, however, that investors have no labor income (i.e., no human capital). When labor income is included in the model of portfolio choice, individuals optimally change their allocations of financial assets in a pattern related to the life cycle. In other words, optimal asset allocation depends on the risk–return characteristics of assets and the flexibility of the individual’s labor income (such as how much or how long the investor works). In our model, the investor adjusts the financial portfolio to compensate for nontradable risk exposures in human capital (Merton 1971; Bodie, Merton, and Samuelson 1992; Heaton and Lucas 1997; Jagannathan and Kocherlakota 1996; Campbell and Viceira 2002). The key theoretical implications of the models that include labor income are as follows: (1) Young investors will invest more in stocks than older investors; (2) investors with safe labor income (thus safe human capital) will invest more of their financial portfolio in stocks; (3) investors with labor income highly correlated with stock markets will invest their financial assets in less risky assets; and

Figure 1. Relationships among Human Capital, Asset Allocation, and Life Insurance

Figure 2. Expected Financial Capital and Human Capital over the Working-Life Cycle
(4) the investor’s ability to adjust his or her labor supply (i.e., higher flexibility) also increases the investor’s allocation to stocks.

Empirical studies show, however, that most investors do not, considering the risk of their human capital, efficiently diversify their financial portfolios. Benartzi (2001) and Benartzi and Thaler (2001) showed that many investors use primitive methods to determine asset allocations and many of them invest very heavily in the stock of the company they work for.3

Life Insurance and Human Capital. Many researchers (e.g., Samuelson; Merton 1969) have pointed out that the lifetime consumption and portfolio decision models need to be expanded to take into account lifetime uncertainty (or mortality risk). Yaari’s 1965 article is considered the first classical paper on this topic. Yaari pointed out ways of using life insurance and life annuities to insure against lifetime uncertainty. He also derived conditions under which consumers would fully insure against lifetime uncertainty.4

Theoretical studies show a clear link between the demand for life insurance and the uncertainty of human capital. Campbell (1980) argued that for most households, labor income uncertainty dominates the uncertainty of financial capital income. He further developed solutions, based on human capital uncertainty, for the optimal amount of insurance a household should purchase.5 Smith and Buser (1983) used mean–variance analysis to model life insurance demand in a portfolio context. They derived optimal insurance demand and the optimal allocation between risky and risk-free assets. They found that the optimal amount of insurance depends on two components: the expected value of human capital and the risk–return characteristics of the insurance contract. Addressing the uncertainties and inadequacies of an individual’s human capital, Ostaszewski (2003) went further by stating that life insurance is the business of human capital securitization. Empirical studies of life insurance adequacy, however (e.g., Auerbach and Kotlikoff 1991), have shown that underinsurance is prevalent. Gokhale and Kotlikoff (2002) argued that questionable financial advice, inertia, and the unpleasantness of thinking about one’s death are the likely causes of underinsurance.

The Model

To merge asset allocation and human capital with optimal demand for life insurance, we need a solid understanding of the actuarial factors that affect the pricing of a life insurance contract. Note that, although there are a number of life insurance product variations—term life, whole life, universal life—each worthy of its own financial analysis, we focus exclusively on the most fundamental type—namely, the one-year, renewable term policy.6 On a basic economic level, a one-year, renewable term policy premium is paid at the beginning of the year—or on the individual’s birthday—and protects the human capital of the insured for the duration of the year. (If the insured person dies within that year, the insurance company pays the face value of the policy to the beneficiaries soon after the death or prior to the end of the year.) For the next year, the contract is guaranteed to start anew, with new premium payments made and protection received; hence, the word “renewable.”

In this general overview of how to think about the joint determination of optimal asset allocation and prudent life insurance holdings, we assume there are two asset classes. The investor can allocate financial wealth between a risk-free asset and a risky asset (i.e., a U.S. government bond and a stock). Also, the investor can purchase a term life insurance contract that is renewable each period. The investor is assumed to make asset allocation and insurance purchase decisions and receive labor income at the beginning of each period. The investor’s objective is to maximize overall utility, which includes utility from the investor’s alive state and the investor’s dead state.

The optimization problem is to determine the amount of life insurance (the face value of life insurance—that is, the death benefit) together with the allocation to risky assets to maximize the end-year utility of total wealth (human capital plus financial wealth) weighted by the alive and dead states subject to certain budget constraints.

The optimization problem in the model can be expressed as

$$\max_{\theta, \alpha} E \left[ (1 - D)(1 - \bar{q}_x) U_{alive} \left( W_{x+1} + H_{x+1} \right) \right]$$

$$+ D \left( \bar{q}_x \right) U_{dead} \left( W_{x+1} + \theta_x \right),$$

where

- $\theta_x$ = amount of life insurance
- $\alpha_x$ = allocation to the risky asset
- $D$ = relative strength of the utility of bequest, as explained in Appendix C
- $\bar{q}_x$ = subjective probabilities of death at the end of year $x + 1$ conditional on being alive at age $x$
- $1 - \bar{q}_x$ = subjective probability of survival
- $W_{x+1}$ = wealth level at age $x + 1$, as explained in Appendix C
- $H_{x+1}$ = human capital

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and $U_{\text{alive}}(\cdot)$ and $U_{\text{dead}}(\cdot)$ are the utility functions associated with the alive and dead states. Appendix C contains a detailed specification of the model and optimization problem.

The model was inspired by Campbell and by Smith and Buser, but we extended their framework in a number of important directions. First, we linked the asset allocation decision with the life insurance purchase decision into one framework by incorporating human capital. Second, we specifically took into consideration the impact of the bequest motive—that is, the investor’s desire to leave a bequest to heirs—on asset allocation and life insurance. Third, we explicitly modeled the volatility of labor income and its correlation with financial market volatility. Fourth, we also modeled the investor’s subjective survival probability.

Human capital is the central component that links the insurance and the investment decisions. An investor’s human capital can be viewed as a “stock” if its correlation with a given financial market subindex is high and the volatility of the person’s labor income is high. It can be viewed as a “bond” if both the correlation and the volatility are low. In between these two extremes, human capital is a diversified portfolio of stocks and bonds, plus idiosyncratic risk. We are quite cognizant of the difficulties involved in calibrating these variables—as pointed out by Davis and Willen (2000); we rely on some of Davis and Willen’s parameters for our numerical examples.

The model has several important implications. First, it clearly shows that asset allocation decisions and life insurance decisions both affect an investor’s overall utility; thus, these decisions should be made jointly. The model also shows that human capital is the central factor. The effects of human capital on asset allocation and life insurance decisions are generally consistent with the existing literature (e.g., Campbell and Viceira; Campbell). One of our major enhancements is the explicit modeling of correlation between the shocks to labor income and financial market returns. The correlation between income and risky asset returns plays an important role in both decisions. All else being equal, as the correlation term between shocks to income and risky assets increases, the optimal allocation to risky assets should decline and so should the optimal quantity of life insurance. Although the first decision might be intuitive from a portfolio theory perspective, we provide precise analytic guidance on how it should be implemented. Furthermore, and contrary to intuition, we show that a higher correlation with any given subindex should reduce the demand for life insurance. The reason is that the higher the correlation, the higher the discount rate used to compute human capital based on future income. A higher discount rate implies a lower valuation of human capital—thus, less insurance demand.

The second implication is that the asset allocation decision affects well-being in both the alive state and the dead (bequest) state whereas the life insurance decision affects primarily the dead state. Bequest preference is arguably the most important factor other than human capital when evaluating life insurance demand. Investors who weight bequest more (higher $D$) are likely to purchase more life insurance.

A unique aspect of our model is the consideration of subjective survival probability ($1 - \tilde{q}$); the model shows the intuitive result that investors with low subjective survival probability will tend to buy more life insurance. This adverse-selection problem is well documented in the insurance literature.

Other implications are consistent with the existing literature. For example, our model implies that (everything else being equal) the greater the financial wealth, the lower the life insurance demand. More financial wealth also indicates a more conservative portfolio when human capital resembles a “bond.” When human capital resembles a “stock,” more financial wealth indicates a more aggressive portfolio. Naturally, risk tolerance also has a strong impact on the asset allocation decision. We found that investors with less risk tolerance will invest conservatively and buy more life insurance.

These implications are illustrated in the case studies presented in the next section.

## Case Studies

To understand the predictions of the model, we analyzed the optimal asset allocation decision and the optimal life insurance coverage for five different cases. We solved the decision problem via simulation; the detailed solving process is presented in Appendix C.

For all five cases, we assumed the investor can invest in two asset classes. Table 1 provides the capital market assumptions used for the cases. We also assumed that the investor is male. His preference for leaving a bequest is one-fourth of his

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preference for consumption in the live state; that is, $1 - D = 0.8$ and $D = 0.2$. He is agnostic about his relative health status (i.e., his subjective survival probability is equal to the objective actuarial survival probability). His income is expected to grow with inflation, and the volatility of the growth rate is 5 percent. His real annual income is $50,000, and he saves 10 percent each year. He expects to receive a pension of $10,000 each year (in today’s dollars) when he retires at age 65. His current financial wealth is $50,000. The investor is assumed to follow constant relative risk aversion (CRRA) utility with a risk-aversion coefficient of $\gamma$. Finally, he rebalances his financial portfolio and renews his term life insurance contract annually. These assumptions remain the same for all cases. Other parameters, such as initial wealth, will be specified in each case.

**Case #1: Human Capital, Financial Asset Allocation, and Life Insurance Demand over Lifetime.** In this case, we assumed that the investor has moderate risk aversion (a CRRA, $\gamma$, of 4). Also, the correlation between shocks to the investor’s income and the market return of the risky asset is 0.20. For a given age, the amount of insurance this investor should purchase can be determined by his consumption/bequest preference, risk tolerance, and financial wealth. His expected financial wealth, human capital, and the derived optimal insurance demand over the investor’s life from age 25 to age 65 are presented in Figure 3.

Several results of modeling this investor’s situation are worth noting. First, human capital gradually decreases as the investor gets older and his remaining number of working years becomes fewer. Second, the amount of his financial capital increases as he ages as a result of growth of his existing financial wealth and the additional savings he makes each year. The allocation to risky assets decreases as the investor ages because of the dynamic between human capital and financial wealth over time. Finally, the investor’s demand for insurance decreases as the investor ages. This result is not surprising because the primary driver of insurance demand is human capital, so the decrease in human capital reduces insurance demand.

**Case #2: Strength of the Bequest Motive.** This case shows the impact of the bequest motive on the optimal decisions about asset allocation and insurance. In the case, we assumed the investor is 45 and has an accumulated financial wealth of $500,000. The investor has a moderate CRRA coefficient of 4. The optimal allocations to the risk-free asset and insurance for various bequest preferences are presented in Figure 4.

In this case, insurance demand increases as the bequest motive strengthens (i.e., as $D$ gets larger). This result is expected because an investor with a
strong bequest motive is highly concerned about his or her heirs and has an incentive to purchase a large amount of insurance to hedge the loss of human capital. In contrast, Figure 4 shows almost no change in the proportional allocation to the risk-free asset at different strengths of bequest motive. This result indicates that asset allocation is primarily determined by risk tolerance, returns on the risk-free and risky assets, and human capital. This case shows that the bequest motive has a strong effect on insurance demand but little effect on optimal asset allocation.\(^\text{15}\)

**Case #3: Effect of Risk Tolerance.** In this case, we again assumed the investor is age 45 and has accumulated financial wealth of $500,000. The investor has a moderate bequest preference level (i.e., \(D = 0.2\)). The optimal allocations to the risk-free asset and the optimal insurance demands for this investor for various risk-aversion levels are presented in Figure 5.

As expected, allocation to the risk-free asset increases with the investor’s risk-aversion level—the classic result in financial economics. Actually, the optimal portfolio is 100 percent in stocks for...
risk-aversion levels less than 2.5. The optimal amount of life insurance follows a similar pattern: Optimal insurance demand increases with risk aversion. For an investor with moderate risk aversion (a CRRA coefficient of 4), optimal insurance demand is about $290,000, which is roughly six times the investor’s current income of $50,000. Naturally, conservative investors should invest more in risk-free assets and buy more life insurance than aggressive investors.

**Case #4: Financial Wealth.** For this case, we held the investor’s age at 45 and the risk preference and the bequest preference at moderate levels (a CRRA coefficient of 4 and bequest level of 0.2). The optimal asset allocations to the risk-free asset and the optimal insurance demands for various levels of financial wealth are presented in Figure 6.

First, Figure 6 shows that the optimal allocation to the risk-free asset increases with initial wealth. This outcome may seem inconsistent with the CRRA utility function because the CRRA utility function implies that the optimal asset allocation does not change with the amount of wealth the investor has. Note, however, that wealth here includes both financial wealth and human capital. In fact, this case is a classic example of the effect of human capital on optimal asset allocation. An increase in financial wealth not only increases total wealth but also reduces the percentage of total wealth represented by human capital. In this case, human capital is less risky than the risky asset. When initial wealth is low, human capital dominates total wealth and asset allocation. As a result, to achieve the target asset allocation of a moderate investor—say, an allocation of 60 percent in the risk-free asset and 40 percent in the risky asset—the closest allocation is 100 percent in the risky asset because human capital is illiquid. With an increase in initial wealth, the asset allocation gradually adjusts until it approaches the asset allocation a moderately risk averse investor desires.

Second, Figure 6 shows that optimal insurance demand decreases with financial wealth. This result can be intuitively explained through the substitution effects of financial wealth and life insurance. In other words, with a large amount of wealth in hand, one has less demand for insurance because the loss of human capital will have much less impact on the well-being of one’s heirs. The optimal amount of life insurance decreases from more than $400,000 when the investor has little financial wealth to almost 0 when the investor has $1.5 million in financial assets.

In summary, for an investor whose human capital is less risky than the stock market, the more substantial the investor’s financial assets are, the more conservative optimal asset allocation is and the smaller life insurance demand is.

**Case #5: Correlation between Shocks to Wage Growth Rate and Stock Returns.** In this case, we want to evaluate the life insurance and asset allocation decisions for investors with highly correlated income and human capital. This kind of correlation can happen when an investor’s income...
is closely linked to the stock performance of the company where the investor works or when the investor’s compensation is highly influenced by the financial market (e.g., the investor works in the financial industry).

Again, the investor’s age is 45 and he has a moderate risk preference and bequest preference. Optimal asset allocation to the risk-free asset and insurance demand for various levels of financial wealth in this situation are presented in Figure 7.

As the correlation between income and stock market return increases, optimal asset allocation becomes more conservative (i.e., more allocation is made to the risk-free asset). One way to look at the reason is that a higher correlation between human capital and the stock market results in less diversification—thus, higher risk—in this investor’s total portfolio (human capital plus financial capital). To reduce this risk, the investor will invest more financial wealth in the risk-free asset.

Optimal insurance demand decreases as the correlation increases. The reason lies with the purpose of life insurance. Because life insurance is purchased to protect the fruits of human capital for the family, as the correlation between the risky asset and the investor’s income flow increases, the ex ante value of human capital to the surviving family decreases. This decreased value of human capital induces lower demand for insurance. Also, less money spent on life insurance indirectly increases the amount of financial wealth the investor can invest, so the investor can invest more in risk-free assets to reduce the risk associated with his total wealth.18

In summary, as wage income and stock market returns become more correlated, optimal asset allocation becomes more conservative and the demand for life insurance falls.

Conclusion

We expanded on the Mertonian idea that human capital is a shadow asset class that is worth much more than financial capital early in life and that it has unique risk and return characteristics. Human capital—even though it is not traded and is highly illiquid—should be treated as part of a person’s endowed wealth that must be protected, diversified, and hedged.

We developed a unified human capital-based framework to help individual investors with life insurance and asset allocation decisions. And we presented five case studies to demonstrate the optimal decisions in different scenarios. The model provided several key results:

- Investors need to make asset allocation decisions and life insurance decisions jointly.
- The magnitude of human capital, its volatility, and its correlation with other assets significantly affect the two decisions over the lifecycle.
- Bequest preferences and a person’s subjective survival probability have significant effects on the person’s demand for insurance but little influence on the person’s optimal asset allocation.
- Conservative investors should invest relatively more in risk-free assets and buy more life insurance.

We demonstrated that the correlation between human capital and financial capital (i.e., whether

![Figure 7. Case #5: Insurance Demand and Asset Allocation by Correlation Level](image-url)
the person’s human capital resembles a bond or a stock) has a noticeable and immediate impact on a person’s demand for life insurance as well as the usual portfolio considerations. The decisions regarding how much life insurance a person needs and where the person should invest cannot be made separately. Rather, they are different aspects of the same problem. For instance, a person whose income relies heavily on commissions should consider his human capital “stock-like” because the income is highly correlated with the market. This characteristic results in great uncertainty in his human capital, so he should purchase less insurance and invest more financial wealth in bonds. Conversely, the human capital of a tenured university professor is more “bond-like,” so she should purchase more insurance and invest financial wealth in stocks.

Obviously, more research remains to be done to make the joint decision making suitable for practical applications. One possible direction would be to model the various types of life insurance and their unique tax-sheltering aspects within a unified asset allocation framework. For example, whole life insurance (and other forms of variable life insurance) could be viewed as a hedge against possible changes in systematic population mortality rates, so it might coexist in an optimal portfolio with short-term life insurance. Another direction would be to diverge from the traditional expected utility models by examining other methods, such as minimizing shortfall probabilities, to determine the appropriate asset allocation/life insurance decision.

Appendix A. Human Capital

Let the symbol $h_t$ denote the random (real, after-tax) wage or salary that a person will receive during the discrete time period (or year) $t$; then, in general, the expected discounted value of this wage flow at current time $t_0$ is represented mathematically by

$$DVHC = \sum_{t=1}^{n} \frac{E[h_t]}{(1 + r + v)^t},$$

(A1)

where $n$ is the number of wage periods over which we are discounting, $r$ is the relevant risk-free discount rate, and $v$ is a (subjective) parameter that captures illiquidity plus any other potential risk premium associated with one’s human capital. In Equation A1, the expectation $E[h_t]$ in the numerator converts the random wage into a scalar. Note that, in addition to expectations (under a physical, real-world measure) in Equation 1, the denominator’s $v$, which accounts for all broadly defined risks, obviously reduces the $t_0$ value of the expression $DVHC$ accordingly.

And depending on the investor’s specific job and profession, he or she might be expected to earn the same exact $E[h_t]$ in each time period $t$, yet the random shocks to wages, $h_t - E[h_t]$, might have very different statistical characteristics vis-à-vis the market portfolio; thus, each profession or job would induce a distinct “risk premium” value for $v$, which would then lead to a lower discounted expected value of human capital. Therefore, because we are discounting with an explicit risk premium, we feel justified in also using the term “financial economic value of human capital” to describe $DVHC$.

Note that when we focus on the correlation or covariance between human capital and other macro-economic or financial factors, we are, of course, referring to the correlation between shocks $h_t - E[h_t]$ and shocks to or innovations in the return-generating process in the market. This correlation can induce a (quite complicated) dependency structure between $DVHC$ in Equation A1 and the dynamic evolution of the investor’s financial portfolio.\(^{19}\)

Appendix B. One-Year, Renewable Term Life Pricing Mechanism

The one-year, renewable term policy premium is paid at the beginning of the year—or on the individual’s birthday—and protects the human capital of the insured for the duration of the year. If the insured person dies within that year, the insurance company pays the face value to the beneficiaries soon after the death or prior to the end of the year. If the insured does not die, the contract is guaranteed to start anew the next year, with new premium payments to be made and protection received.

The policy premium is obviously an increasing function of the desired face value, and the two are related by the simple formula:

$$P = \frac{q}{1 + r} \theta;$$

(B1)

that is, premium $P$ is calculated by multiplying the desired face value, $\theta$, by the probability of death, $q$, and then discounting by the interest rate factor, $1 + r$.

The theory behind Equation B1 is the well-known law of large numbers, which guarantees that probabilities become percentages when individuals are aggregated. Note the implicit assumption in Equation B1 that, although death can occur at any time during the year (or term), the premium payments are made at the beginning of the year and the face values are paid at the end of the year. From the insurance company’s perspective, all of the premiums received from the group of $N$ individuals with the same age (i.e., probability of death $q$) and with contracts having face value $\theta$ are commingled and
invested in an insurance reserve earning rate of interest \( r \), so at the end of the year, \( PN(1 + r) \) is partitioned among the \( qN \) beneficiaries.

No savings component or investment component is embedded in the premium defined by Equation B1. Rather, at the end of the year, the survivors lose any claim to the pool of accumulated premiums because all funds go directly to the beneficiaries.

As the individual ages and probability of death increases (denoted by \( q_x \)), the same exact face amount (face value) of life insurance, \( \theta \), will cost more and will induce a higher premium, \( P_x \), as per Equation B1. In practice, the actual premium is loaded by an additional factor, denoted by \( 1 + \lambda \) to account for commissions, transaction costs, and profit margins; so, the actual amount paid by the insured is closer to \( P(1 + \lambda) \), but the underlying pricing relationship, driven by the law of large numbers, remains the same.

Also, from the perspective of traditional financial planning, the individual conducts a budgeting analysis to determine his or her life insurance demands (i.e., the amount the surviving family and beneficiaries need to replace the lost wages in present-value terms). That quantity would be taken as the required face value in Equation B1, which would then lead to a premium. Alternatively, one can think of a budget for life insurance purchases, in which case, the face value would be determined by Equation B1.

In our model and the discussion, we “solve” for the optimal age-varying amount of life insurance, denoted by \( \theta_x \), which then induces an age-varying policy payment, \( P_x \), which maximizes the welfare of the family by taking into account risk preferences and attitudes toward bequest.

Appendix C. Model Specification: Optimal Asset Allocation and Insurance Demands

In this model, the investor is currently age \( x \) and will retire at age \( Y \). The term “retirement” is simply meant to indicate that the human capital income flow is terminated and the pension phase begins. The assumption is that the financial portfolio will be rebalanced annually and that the life insurance—which is assumed to be of the one-year term variety—will be renewed annually. Taxes are not considered in the model.

The investor would like to know how much (i.e., the face value of term life) insurance he should purchase and what fraction of his financial wealth he should invest in risky assets (stock).

In the model, an investor determines the amount of life insurance demand, \( \theta_x \)—the face value of life insurance (that is, the death benefit)—together with allocation \( \alpha_x \) to risky assets to maximize end-year utility of his total wealth (human capital plus financial wealth) weighted by the “alive” and “dead” states. The optimization problem is expressed as

\[
\max_{\theta_x,\alpha_x} E \left[ (1-D)(1-\tilde{q}_x) U_{alive} (W_{x+1} + H_{x+1}) \right. \\
+ D(\tilde{q}_x) U_{dead} (W_{x+1} + \theta_x) \right],
\]

which is the same as Equation 1 in the text, subject to the following budget constraint:

\[
W_{x+1} = \left[ W_x + h_x - (1+\lambda)q_x \theta_x e^{-r_y} - C_x \right] \\
\times \left[ \mu_x e^{-\frac{1}{2}\sigma_x^2} + (1-\alpha_x) e^{r_y} \right],
\]

where \( e \) is the exponent, 2.7182,

\[
\theta_0 \leq \theta_x \leq \frac{(W_x + h_x - C_x)e^{r_y}}{(1+\lambda)q_x},
\]

and

\[
0 \leq \alpha_x \leq 1.
\]

The symbols, notations, and terminology used in the optimal problem are as follows:

- \( \theta_x \) = amount of life insurance.
- \( \alpha_x \) = allocation to risky assets.
- \( D \) = relative strength of the utility of bequest. Individuals with no utility of bequest will have \( D = 0 \).
- \( q_x \) = objective probability of death at the end of the year \( x + 1 \) conditional on being alive at age \( x \). This probability is determined by a given population (i.e., mortality table).
- \( \tilde{q}_x \) = subjective probability of death at the end of the year \( x + 1 \) conditional on being alive at age \( x \); \( 1 - \tilde{q}_x \) denotes the subjective probability of survival. The subjective probability of death may be different from the objective probability. In other words, a person might believe he or she is healthier (or less healthy) than average. This belief would affect expected utility but not the pricing of the life insurance, which is based on an objective population survival probability.
- \( \lambda \) = fees and expenses (i.e., actuarial and insurance loading) imposed and charged on a typical life insurance policy.
- \( W_t \) = financial wealth at time \( t \). The market has two assets, one risky and one risk free. This assumption is consistent with the two-fund separation theorem of traditional portfolio theory. Of course, the approach could be expanded to multiple asset classes.
\[ rf = \text{return on the risk-free asset.} \]

\[ S = \text{value of the risky asset. This value follows a discrete version of a geometric Brownian motion:} \]

\[ S_{t+1} = S_t \exp \left( \mu_S - \frac{\sigma^2}{2} + \sigma S Z_{S,t+1} \right), \quad (C5) \]

where \( \mu_S \) is the expected return, \( \sigma_S \) is the standard deviation of return of the risky asset, \( Z_{S,t} \) is an independent random variable, and \( Z_{S,t+1} \sim N(0,1) \).

\[ h_t = \text{labor income. In our numerical cases, we assumed} \]

\[ h_t = h_t \exp(\mu_t + \sigma_t Z_{h,t+1}), \quad (C6) \]

where, \( h_t > 0; \mu_t \) and \( \sigma_t \) are, respectively, the annual growth rate and the annual standard deviation of the income process; \( Z_{h,t} \) is an independent random variable; and \( Z_{h,t+1} \sim N(0,1) \).

Based on Equation C6, for a person at age \( x \), income at age \( x + t \) is determined by

\[ h_{x+t} = h_t \prod_{k=1}^{t} \exp(\mu_t + \sigma_t Z_{h,k}), \quad (C7) \]

The correlation between shocks to labor income and the return of the risky asset is \( \rho \), and

\[ Z_h = \rho Z_S + \sqrt{1-\rho^2} Z, \quad (C8) \]

where \( Z \) is a standard Brownian motion independent of \( Z_S \); that is,

\[ \text{cor}(Z_S, Z_h) = \rho. \quad (C9) \]

\[ H_t = \text{present value of future income from age} \]

\[ H_t \text{ to death. Income after retirement is the} \]

\[ Y-x \]

\[ = \sum_{j=t+1}^{Y-x} \left( h_{x+j} \exp \left[ -(j-t)(r_f + \eta_h + \zeta_h) \right] \right), \quad (C10) \]

where \( \eta_h \) is the risk premium (discount rate) for the income process and captures the market risk of income; \( \zeta_h \) is a discount factor in human capital evaluation to account for the illiquidity risk associated with one’s job. In the numerical examples, we assumed a 4 percent discount rate per year.\(^{20}\)

Based on the CAPM, \( \eta_h \) can be evaluated by

\[ \eta_h = \frac{\text{cov}(Z_{h,t}, Z_S)}{\text{var}(Z_S)} \left[ \mu_S - (r_f - 1) \right], \quad (C11) \]

Furthermore, the expected value of \( H_t, E(H_{x+t}) \), is defined as the human capital a person has at age \( x + t + 1 \).

\[ C_t = \text{consumption in year} \]

\[ t. \]

\[ U(x) = \frac{x^{1-\gamma}}{1-\gamma} \quad (C12) \]

for \( x > 0 \) and \( \gamma \neq 1 \) and

\[ U(x) = \ln(x) \quad (C13) \]

for \( x > 0 \) and \( \gamma = 1 \). The power utility function is used in the examples for both \( U_{\text{alive}}(\cdot) \) and \( U_{\text{dead}}(\cdot) \), which are the utility functions associated with, respectively, the alive and dead states.\(^{21}\)

We solved the problem via simulation. We first simulated the values of the risky asset by using Equation C5. Then, we simulated \( Z_h \) from Equation C8 to take into account the correlation between income change and the return of the financial market. Finally, we used Equation C6 to generate income over the same period. Human capital, \( H_{x+t} \), is calculated by using Equations C7 and C10. If wealth level at age \( x + 1 \) is less than 0, we set the wealth equal to 0; that is, we assumed the investor has no remaining financial wealth. We simulated this process \( N \) times. The objective function was evaluated by using

\[ \frac{1}{N} \sum_{n=1}^{N} U_{\text{alive}}(W_{x+1}(n) + H_{x+1}(n)) \quad (C14) \]

and

\[ \frac{1}{N} \sum_{n=1}^{N} U_{\text{dead}}(W_{x+1}(n) + \theta_x). \quad (C15) \]

In the numerical examples, we set \( N \) equal to 20,000.
Notes

1. How much an investor should consume or save is an important decision that is also frequently tied to the concept of human capital. This article focuses on the asset allocation and life insurance decisions, however, so we have simplified the model by assuming that the investor has already decided how much to consume/save. And for a model that optimizes consumption in addition to insurance and asset allocation, see Huang, Milevsky, and Wang (2005).

2. The term “human capital” can convey a number of different—at times, conflicting—concepts in the literature. We define human capital here to be the financial economic value of all future wages, which is a scalar quantity and depends on a number of subjective or market equilibrium factors. Appendix A provides a detailed explanation of human capital and the discounted present value of future salaries.

3. Heaton and Lucas (2000) showed that wealthy households with high and variable business income invest less in the stock market than other, similar wealthy households, which is consistent with the theoretical prediction.

4. Like Yaari, Fischer (1973) pointed out that these earlier models either dealt with an infinite horizon or took the date of death to be known with certainty.

5. Economides (1982) argued that in a corrected model, Campbell’s approach underestimates the optimal amount of insurance coverage. Our model takes this correction into consideration.

6. Appendix B provides a description of the pricing mechanism of the one-year, renewable term life insurance policy. We believe that all other types of life insurance policies are financial combinations of term life insurance with investment accounts, added tax benefits, and embedded options—although investigating this aspect is beyond the scope of this article.

7. Bernheim (1991) and Zietz (2003) showed that the bequest motive has a significant impact on life insurance demand.

8. Note that when we make statements such as: “This person’s human capital is 40 percent long-term bonds, 30 percent financial services, and 30 percent utilities,” we mean that the unpredictable shocks to future wages have a given correlation structure with these particular subindices. Thus, for example, the human capital of a tenured university professor could be considered to be a 100 percent real-return (inflation-linked) bond because shocks to wages—if there are any—would not be linked to any financial subindex.

9. The scenarios in which asset allocation and life insurance decisions are not linked are when the investor derives her or his utility 100 percent from consumption or 100 percent from bequest. Both are extreme scenarios, especially the 100 percent from bequest. A well-designed questionnaire could help elicit an individual’s attitude toward the importance of bequest, even though a precise estimate may be hard to obtain.

10. Actuarial mortality tables can be taken as a starting point for this factor. Life insurance is already priced to take into account adverse selection.

11. The salary growth rate and the volatility were chosen mainly to show the implications of the model. They are not necessarily representative.

12. The mortality and insurance fees and expenses imposed for handling the insurance contract were assumed to be 12.5 percent.

13. Davis and Willen estimated the correlation between labor income and equity market returns by using the U.S. Department of Labor’s Current Occupation Survey. They found that the correlation between equity returns and labor income typically lies in the interval from –0.10 to 0.20.

14. In our model, subjective survival probability has a similar impact on optimal insurance need and asset allocation as the bequest motive does. When subjective survival probability is high, the investor will buy less insurance.

15. This result is close to the typical recommendation made by financial planners (i.e., purchase a term life insurance policy that has a face value four to seven times one’s current income). See, for example, Todd (2004).

16. In this case, income has a real growth rate of 0 percent and a standard deviation of 5 percent, yet the expected real return on stock is 8 percent with a standard deviation of 20 percent.

17. See Case #3 for a detailed discussion of the wealth impact.

18. For a more rigorous and mathematically satisfying treatment of the ongoing interaction between human capital and market returns as it pertains to the purchase of life insurance in a continuous-time framework, see Huang, Milevsky, and Wang (2005).

19. The 4 percent discount rate translates into about a 25 percent discount on the overall present value of human capital for a 45-year-old person with 20 years of future salary. This 25 percent discount is consistent with empirical evidence on the discount factor between restricted stocks and their unrestricted counterparts (see Amihud and Mendelson 1991). Also, Longstaff (2002) reported that the liquidity premium for the longer-maturity U.S. T-bond is 10–15 percent of the value of the bond.

20. Stutzer (2004) pointed out the difficulties in applying expected utility theories in practice and proposed using the minimization of short-fall probability as an alternative. We followed the traditional expected utility model of linking asset allocation and portfolio decisions to individual risk aversion.

References


