Morningstar Indexes Distance to Default Methodology Paper

Morningstar Indexes
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Black-Scholes-Merton Structural Model Governing Equations

We may use equity prices and fundamentals to estimate the default probability of a company. It has been shown (Merton 1974) that the company's equity may be modeled as an option on the assets of the company. We may then use the solution to the Black-Scholes-Merton equation for a European call option (1-4) to value the equity. This yields a structural model that relates the equity value to the value of the unobservable asset values.

1. $E_0 = D_0 + A_0\Phi(d_1) - L_T\, e^{-r_f T}\,\Phi\,(d_2)$

Where

2. $\Phi_{(X)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2}\, dz$

And

3. $d_1 = \frac{\ln\frac{A_0 - D_0}{L_T} + \left(r_f + \sigma_A^2/2\right)T}{\sigma_A\sqrt{T}}$

4. $d_2 = d_1 - \sigma_A\sqrt{T} = \frac{\ln\frac{A_0 - D_0}{L_T} + \left(r_f - \sigma_A^2/2\right)T}{\sigma_A\sqrt{T}}$

The distance to default is d_2 and the risk-neutral probability that the company will default on the debt is $\Phi\,(-d_2)$. For our purposes, we will assume a constant time horizon of one year, setting T=1.

Data point definitions used as inputs; all are in local currencies:

E_0 = the current value of the aggregated company equity.

D_0 = total dividends to be paid within one year using forward dividend, else historical dividends.

L_T = liabilities due within one year using short-term liabilities, else long-term liabilities

r_f = logarithmic risk-free rate (annual rate).

Data Points That Are Calculated or Solved For

σ_E = standard deviation of logarithmic equity returns (annual) (instantaneous value). This is accomplished by using the exponentially weighted moving average, or EWMA, estimate.

A_0 = the current value of the firm's assets in local currency.

σ_A = standard deviation of logarithmic asset returns (annual) (assumed to be constant).

Exponentially Weighted Moving Average Estimation of Equity Volatility

Equity volatility, σ_E, is the most important parameter in the Black-Scholes-Merton model. Therefore, it is important to have an estimation methodology that responds quickly to recently observed changes in the volatility of equity returns. Exponentially weighted averaging is a standard method of achieving this.
Let P_t denote the closing share price on day t and $r_t$denote the daily logarithmic return on equity. We have:

5. $r_t = \ln(P_t) - \ln(P_{t-1})$

Let λ denote the exponential decay rate. We set λ to value in the range 0.97. We calculate the exponentially weighted average daily logarithmic return recursively as follows:

6. $\mu_{1t} = (1-\lambda)\, r_t + \lambda\mu_{1t-1}$

We initialize μ10=0.

Similarly, we calculate the exponentially weighted average daily logarithmic return squared recursively as follows:

7. $\mu_{2t} = (1-\lambda)\, \mathrm{r}_t^2 + \lambda\mu_{2t-1}$

We initialize μ20 to an initial estimate of σ_E^2/NTY, NTY being the assumed number of trading days per year, which we take to be 252.

Our estimate of the annualized standard deviation of logarithmic equity returns described below on day t is:

8. $\hat{\sigma}_{Et} = \sqrt{NTY\,(\mu_{2t} - \mu_{1t}^2)}$

Solve for Asset Value and Standard Deviation of Log Asset Returns

We now would like to solve for the asset value and asset volatility. To do this, we need another equation to solve for these two variables. Luckily, we may introduce another equation via Ito's Lemma to solve the set of nonlinear equations that implicitly define the asset value and volatility.

9. $\sigma_E \mathrm{E}_0 = \frac{\partial E_0}{\partial A_0} A_0 \sigma_A = \Phi(d_1)\, A_0 \sigma_A$

Equations (1-4, 9) must now be solved. This may be done by recasting the problem as an optimization problem.
Let us write the zero set of equations (1) and (9) as:

10. $G(A_0\,, \sigma_A\,) = E_0 - D_0 - A_0\,\Phi\,(d_1) + L_T e^{-r_f T} \Phi(d_2)$

11. $H(A_0\,, \sigma_A\,) = \sigma_E E_0 - \Phi\,(d_1)\,A_0\sigma_A$

This set of equations can be solved for by using a constrained optimization procedure (we use Nelder-Mead) using the following objective function:

12. $\min_{A_0\,,\sigma_A} F(A_0\,, \sigma_A) = G^2 + H^2$

Use Values to Calculate Distance to Default and Probability of Default

Once we have solved for the previous values, we can calculate the distance to default:

13. $d_2 = d_1 - \sigma_A\sqrt{T} = \dfrac{\ln\frac{A_0 - D_0}{L_T} + \left(r_f - {\sigma_A^2}/{2}\right)T}{\sigma_A\sqrt{T}}$

And the probability of default is:

14. $\Phi\,(-d_2)$

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