When the Wright Brothers pioneered powered flight in 1903, their genius lay in conquering the three axes of control: pitch, yaw, and roll. Over the years, technologies advanced, planes crashed, and aviation evolved to compensate. By 1952, the Wright’s original airplane was barely recognizable in a world of jets and even supersonic aircraft, which were nonetheless still governed by the same three principles of control.

In 1952, another pioneer, Harry Markowitz, invented portfolio optimization. His genius was also based on three principles: risk, reward, and the correlation of assets in a portfolio. Over the years, technologies advanced and markets crashed, but the portfolio-optimization models used by many investors did not evolve to compensate. This is surprising in light of the fact that Markowitz himself was a pioneer of technological advancement in the field of computational computer science. Furthermore, he did not stand idly by in the area of portfolio modeling, but continued to make improvements in his own models and to influence the models of others. Few of these improvements, however, were picked up broadly in practice.

Going Supersonic
Because Markowitz’s first effort was so simple and powerful, it attracted a great number of followers. The greater the following became, the fewer questioners debated its merits. Markowitz’s original work is synonymous with Modern Portfolio Theory and has been taught in business schools for generations and, not surprisingly, is still widely used today.

Then came the crash of 2008, and at last people are starting to ask questions. The confluence of the recent economic trauma and the technological advances of the past few decades make today the perfect time to describe the supersonic models that can be built around Markowitz’s fundamental principles of risk, reward, and correlation. In a recent paper, we assert that Markowitz’s original work remains the perfect framework for applying the latest in economic thought and technology. We dub our updated model “Markowitz 2.0.”

Markowitz 2.0
The Flaw of Averages
The 1952 mean-variance model of Harry Markowitz was the first systematic attempt to cure what Savage [2009] calls the “flaw of averages.” In general, the flaw of averages is a set of systematic errors that occur when people use single numbers (usually averages) to describe uncertain future quantities. For example, if you plan to rob a bank of $10 million and have one chance in 100 of getting away with it, your average take is $100,000. If you described your activity beforehand as “making $100,000,” you would be correct on average. But this is a terrible characterization of a bank heist. Yet as Savage [2009] discusses, this very “flaw of averages” is made all the time in business practice, and helps explain why everything is behind schedule, beyond budget, and below projection, and was an accessory to the economic catastrophe that culminated in 2008.

Harry Markowitz’s 1952 mean-variance model attempted to cure the flaw of averages by distinguishing between different investments with the same average (expected) return, but with different risks, measured as variance or its square root, standard deviation. This was a breakthrough at the time that ultimately garnered a Nobel Prize for its inventor. However, the use of standard deviation and covariance introduces a higher-order version of the flaw of averages, in that these concepts are themselves a version of averages. CONTINUED ON NEXT PAGE
Adding Afterburners to Traditional Portfolio Optimization

By taking advantage of the very latest in economic thought and computer technology, we can, in effect, add afterburners, or more thrust, to the original framework of the Markowitz portfolio-optimization model. The result is a dramatically more powerful model that is more aligned with 21st century investor concerns, markets, and financial instruments such as options.

Traditional portfolio optimization, commonly referred to as mean-variance optimization, or MVO, suffers from several limitations that can easily be addressed with today's technology. Our discussion here will focus on five practical enhancements:

1. First, we use a scenario-based approach to allow for “fat-tailed” distributions. Fat-tailed return distributions are not possible within the context of traditional mean-variance optimization, where return distributions are assumed to be adequately described by mean and variance.

2. Second, we replace the single-period expected return with the long-term forward-looking geometric mean (GM), as this takes into account accumulation of wealth.

3. Third, we substitute Conditional Value at Risk (CVaR), which only looks at tail risk, for standard deviation, which looks at average variation.

4. Fourth, the original Markowitz model used a covariance matrix to model the distribution of returns on asset classes; we replace this with a scenario-based model that can be generated with Monte Carlo simulation and can incorporate any number of distributions.

5. Finally, we exploit new statistical technologies pioneered by Sam Savage in the field of Probability Management. Savage invented a new technology called the Distribution String, or DIST™, which encapsulates thousands of trials as a single data element or cell, thus eliminating the main disadvantage of the scenario-based approach—the need to store and process large amounts of data.

The Scenario Approach

One of the limitations of the traditional mean-variance optimization framework is that it assumes that the distribution of returns of the assets in the optimization can be adequately described simply by mean and variance alone. The most common depiction of this assumption is to draw the distribution of each asset class as a symmetrical bell-shaped curve. However, as illustrated in Exhibit 1, the return distributions of different asset classes don’t always follow a symmetrical bell-shaped curve. Some assets have distributions that are skewed to the left or right, while others have distributions that are skinnier or fatter in the tails than others.

Over the years, various alternatives have been put forth to replace mean-variance optimization with an optimization framework that takes into account the non-normal features of return distributions. Some researchers have proposed using distributions curves that exhibit skewness and kurtosis (that is, have fat tails) while others have proposed using large numbers of scenarios based on historical data or Monte Carlo simulation.

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Contemplated on Next Page
The scenario-based approach has two main advantages over a distribution curve approach: (1) it is highly flexible; for example, nonlinear instruments such as options can be modeled in a straightforward manner, and (2) it is mathematically manageable; for example, portfolio returns under the scenarios are simply weighted averages of asset-class returns within the scenarios. In this way, the distribution of a portfolio can be derived from the distributions of the asset classes without working complicated equations that might lack analytical solutions; only straightforward portfolio arithmetic is needed.

In standard scenario analysis, there is no precise graphical representation of return distributions. Histograms serve as approximations such as those shown in Exhibit 1. We augment the scenario approach by employing a smoothing technique so that smooth curves represent return distributions. For example, Exhibit 2 shows the distribution curve of annual returns of Large Company Stocks under our approach. Comparing Exhibit 2 with the Large Company Stock histogram in Exhibit 1, we can see that the smooth distribution curve retains the properties of the historical distribution while showing the distribution in a more esthetically pleasing and precise form. Furthermore, our model makes it possible to bring all of the power of continuous mathematics previously enjoyed only by models based on continuous distributions to the scenario approach.

In Exhibit 2, the green line curve is what we get when we use mean-variance analysis and assume that returns follow a lognormal distribution. The blue line is what we get when we use our smoothed scenario-based approach. The area under the blue solid line to the left of the vertical segment shows that the 5th percentile return under our model is −25.8 percent, meaning there is 5% probability of a return of less than −25.8 percent. However, under the lognormal model, the probability of the return being less than −25.8 percent is only 1.6 percent. This illustrates how a mean-variance model can woefully underestimate the probability of tail events.

As Kaplan et al. [2009] discuss, tail events have occurred often throughout the history of capital markets all over the world. Hence, it is important for asset-allocation models to assign nontrivial probabilities to them.

**Geometric Mean versus Single Period Expected Return**

In MVO, reward is measured by expected return, which is a forecast of arithmetic mean. However, over long periods of time, investors...
are not concerned with simple averages of return; rather, they are concerned with the accumulation of wealth. We use forecasted long-term geometric mean (GM) as the measure of reward because investors who plan on repeatedly reinvesting in the same strategy over an indefinite period would seek the highest rate of growth for the portfolios as measured by geometric mean.

**Conditional Value at Risk versus Standard Deviation**

As for risk, much has been written about how investors are not concerned merely with the degree of dispersion of returns (as measured by standard deviation), but rather with how much wealth they could lose. A number of “downside” risk measures have been proposed to replace standard deviation as the measure of risk in strategic asset allocation. While any one of these could be used, our preference is to use Conditional Value at Risk (CVaR).

CVaR is related to Value at Risk (VaR). VaR describes the left tail in terms of how much capital can be lost over a given period of time. For example, a 5% VaR answers a question of the form: Having invested $10,000, there is a 5% chance of losing $X or more in 12 months. (The “or more” implications of VaR are sometimes overlooked by investors, with serious implications.) Applying this idea to returns, the 5% VaR is the negative of the 5th percentile of the return distribution. For example, the 5th percentile of the distribution shown in Exhibit 2 is –25.8% so its 5% VaR is 25.8%. This means there is a 5% chance of losing $2,850 or more on a $10,000 investment. CVaR is the expected or average loss of capital should VaR be breached. Therefore CVaR is always greater than VaR. For example, the 5% CVaR for the distribution shown in Exhibit 2 is 35.8%, or $3,580, on a $10,000 investment.

**Scenarios versus Correlation**

In mean-variance analysis, the covariation of the returns of each pair of asset classes is represented by a single number, the correlation coefficient. This is mathematically equivalent to assuming that a simple linear regression model is an adequate description of how the returns on the two asset classes are related. In fact, the R-square statistic of a simple linear regression model for two series of returns is equal to the square of the correlation coefficient.
However, for many pairs of asset classes, a linear model misses the most important features of the relationship. For example, during normal times, non-U.S. equities are considered to be good diversifiers for U.S. equity investors. But during global crises, all major equity markets move down together. Furthermore, suppose that the returns on two asset classes indexes were highly correlated, but instead of including direct exposures to both in the model, one was replaced with an option on itself. Instead of having a linear relationship, we now have a nonlinear relationship that cannot be captured by a correlation coefficient.

Fortunately, these sorts of nonlinear relationships between returns on different investments can be handled in a scenario-based model. For example, in scenarios that represent normal times, returns on different equity markets could be modeled as moving somewhat apart from each other while scenarios that represent global crises could model the markets as moving downward together.

**Ultrasonic Statistical Technology**

Because it may take thousands of scenarios to adequately model return distributions, until recently, a disadvantage of the scenario-based approach has been that it requires large amounts of data to be stored and processed. Even with the advances in computer hardware, the conventional approach of representing scenarios with large tables of explicit numbers remained problematic.

The phenomenal speed of computers has given rise to the field of Probability Management, an extension of data management to probability distributions rather than numbers. The key component of Probability Management is the Distribution String, or DIST™, which can encapsulate thousands of trials as a single data element. The use of DISTs greatly saves on storage and speeds up processing time, so that a Monte Carlo simulation consisting of thousands of trials can be performed on a personal computer in an instant. While not all asset-management organizations are prepared to create the DISTs needed to drive the GM-CVaR optimization described in Kaplan and Savage [2009], some outside vendors, such as Morningstar Ibbotson, can fulfill this role.

Another facet of Probability Management is interactive simulation technology, which can run thousands of scenarios through a model before the sound of your finger leaving the <Enter> key reaches your ear. These supersonic models allow much deeper intuition into the sensitivities of portfolios, and encourage the user to interactively explore different portfolios, distributional assumptions, and potential black swans. A sample of such an interactive model will be available for download from www.ProbabilityManagement.org in 2010.

**Finale: The New Efficient Frontier**

Putting it all together, we form an efficient frontier of forecasted geometric mean and Conditional Value at Risk as shown in Exhibit 3, (Page 4) incorporating our scenario approach to covariance and new statistical technology. We believe that this efficient frontier is more relevant to investors than the traditional expected return versus standard deviation frontier of MVO because it shows the trade-off between reward and risk that is meaningful to investors; namely, long-term potential growth versus short-term potential loss.

**Learn More**

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**References**


