

# Quant Corner: Updating Monte Carlo Simulation for the 21st Century

## How new technology presents a solution to investment forecasting.



by  
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In 1946, Stanislaw Ulam, a Poland-born mathematician and member of the Manhattan Project, was whiling away the time during his recovery from an illness by playing solitaire and began to wonder about the likelihood of success. So, he stopped playing with the cards and returned to his profession of mathematics to try to calculate the percentage of successful games out of all possible shuffles. This turned out to be harder than he thought. So, he came up with an alternative method using the power of an early computer to simulate 100 card shuffles and then simply count the number of winning hands.

Thus was born a computational technique now known as Monte Carlo Simulation, so named because the basic building block was none other than a computerized version of a roulette wheel with many billions of numbers around the edge. Although it took decades to work out all the kinks in the computerized roulette wheel, Monte Carlo Simulation has become a standard tool of risk management. Its latest incarnations offer several bold advances.

### An Early Application of Monte Carlo Simulation to Asset Allocation

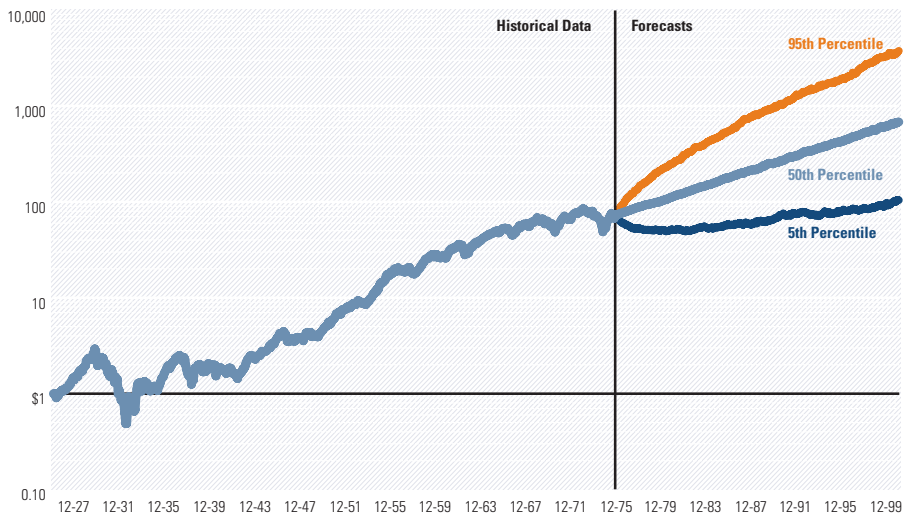
In 1976, Roger Ibbotson, then an assistant professor at the University of Chicago, and Rex Sinquefeld published a paper in the *Journal of Business* called "Stocks, Bonds, Bills, and

Inflation: Simulations of the Future (1976–2000)" as a companion piece to their historical study of asset class returns. In "Simulations," they used the Monte Carlo method developed by Ulam to make probabilistic predictions of the form "there is an X percent chance that \$1 invested in the portfolio will grow to \$Y or more in Z years." Putting together past history with the forecasts, they generated "tulip" or "fan" charts similar to Exhibit 1.

Like Harry Markowitz's 1952 mean-variance model, the Ibbotson-Sinquefeld simulation model was an early attempt to cure what Savage has dubbed the "flaw of averages."<sup>1</sup> In general, the flaw of averages is a set

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Exhibit 1: Ibbotson-Sinquefeld Simulation Chart



of systematic errors that occurs when people use single numbers (usually averages) to describe uncertain future quantities. For example, if you plan to rob a bank of \$10 million and have one chance in 100 of getting away with it, your average take is \$100,000. If you described your activity beforehand as “making \$100,000,” you would be correct, on average. But this is a terrible characterization of a bank heist! Yet this very mistake is made all the time in business practice. It helps explain why everything is behind schedule, beyond budget, and below projections, and it was an accessory to the economic catastrophe that culminated in 2008.

Ibbotson and Sinquefeld simulated each future month’s return on a portfolio from historical monthly returns over the period 1926–74, a period of 588 months. Like Ulam, Ibbotson and Sinquefeld used a computer program to “spin” a roulette wheel with 588 spots 300 times for each simulated future. By running only a few thousand possible futures, they were able to complete the calculations on a mainframe computer of the era in time for publication.

### **Ibbotson and Sinquefeld “Made Easy”**

While there was interest in the Ibbotson-Sinquefeld simulation model at the time of its publication, the technology for running a Monte Carlo Simulation was not readily available to many in the investment community. But four years later in 1980, four researchers published a paper in the *Journal of Business* that showed that, to a large degree, the results of the Ibbotson-Sinquefeld simulations could be replicated without Monte Carlo Simulation<sup>2</sup>. Titled “The Ibbotson-Sinquefeld Simulation Made Easy,” this paper showed that by making a number of simplifying assumptions and applying the Central Limit Theorem<sup>3</sup>, probabilistic forecasts of cumulative wealth can be made using mathematical formulas<sup>4</sup>.

The “Made Easy” model became the standard method for probabilistic forecasting and is in wide use today.

However, as powerful as the “Made Easy” model is, it is not up to the task of forecasting problems other than simple wealth accumulation with no inflows or outflows. Consider the problem of forecasting how long a retiree can make a given amount of wealth last before going broke, assuming that she invests her unspent wealth in a portfolio of risky assets. If we were to assume a fixed rate of return on investments during retirement and solve for the year in which the retiree runs out of money, we would run afoul of the Flaw of Averages because there are many plausible scenarios in which poor returns in the early years cause the retiree to go broke well before the time forecasted. Except under highly simplified assumptions, the only practical way to approach this problem is Monte Carlo Simulation<sup>5</sup>. Hence, the Monte Carlo approach has become the most common method for modeling drawing down wealth during retirement.

Furthermore, the capital markets do not always behave in the way that the simplified models assume. As Kaplan discusses, history is replete with “fat tail” events that are captured by models based on the bell curve (as all of the simplified models are)<sup>6</sup>. This is another reason why Monte Carlo Simulation is usually the most practical approach to investment forecasting.

This is not to say that Monte Carlo Simulation is a silver bullet. There are a number of practical issues when implementing Monte Carlo model that must be taken into consideration. Michele Gambera summarizes a number of these issues<sup>7</sup>, namely:

- 1 The accuracy of the results is limited by the number of simulated histories. Hence, there is a trade-off between the accuracy of the model and the time it takes to run it.
- 2 The amount of time needed to run enough simulated histories might be too long to be practical to obtain enough accuracy to make the model useful.
- 3 The amount of computer storage needed to run a model might be impractically large. For example, to store 1,000 simulated histories over a 25-year period of monthly returns requires storing 300,000 numbers per asset class.

### **A 21st-Century Update**

Fortunately, 21st-century technology addresses these issues not only making Monte Carlo Simulation practical, but also interactive and highly flexible. This is due to three computer technologies that have recently come together—Interactive Simulation, the DIST Distribution String, and Cloud Computing:

#### **Interactive Simulation**

The central processing unit in today’s iPhone is hundreds of times more powerful than the machine used by Ibbotson and Sinquefeld and many times faster than in 2002, the date of Gambera’s publication. Furthermore, several recent software breakthroughs have focused specifically on the speed of Monte Carlo Simulation. Risk Solver Platform, for example, from Nevada-based Frontline Systems, can simulate 100,000 spins of the roulette wheel in Microsoft Excel before the user’s finger has left the Enter key of his computer. The resulting “interactive” simulation provides a new level of intuition into uncertainty. And more speed is on the way. Not only are CPUs getting faster, but machines are being fitted with parallel processors. Many applications cannot be programmed to take advantage of multiple processors. Monte Carlo Simulation is a notable exception and is known in the trade as “embarrassingly parallel.” It may not be long before specialized machines are developed for the sole purpose of running simulations.

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**Exhibit 2: The DIST Distribution String**

...0.756, 0.927, 0.873, 0.253, 0.070, 0.705, 0.665, 0.427,  
0.375, 0.708, 0.733, 0.769, 0.405...Thousands of Scenarios...  
0.094, 0.715, 0.829, 0.012, 0.848, 0.008, 0.024, 0.174,  
0.745, 0.253, 0.242, 0.273, 0.517, 0.446, 0.630, 0.564,  
0.929, 0.129, 0.285, 0.615, 0.629, 0.188, 0.253, 0.745...

	A	B	C	D	E	F	G
1							
2							
3			<b>&lt;dist name="Large Cap" avg="7.02855199E-002"</b>				
4							

**The DIST Distribution String**

The Distribution String is a new standard for packaging thousands of Monte Carlo scenarios into a single data element. (See Exhibit 2.) It was developed by Savage, in collaboration with Oracle Corp., SAS Institute, and Frontline Systems, and along with interactive simulation, addresses the issues raised by Gambera. If interactive simulation is a new light bulb for illuminating uncertainty, then the Distribution String is the AC current that lights the bulb. The 300,000 data elements required to store a 25-year simulation is reduced to 300 DIST elements. And when people say that size does not matter, this does not apply to factors of 1,000.

**Cloud Computing**

The DIST standard is so compact that thousands of Monte Carlo trials may be downloaded over the Web in seconds. This provides a collaborative network in which specialists in financial statistics can produce probability distributions, for immediate consumption by a wide array of investors, worldwide. Hence, it may unleash an industry in the distribution of distributions.

**Implications for Tomorrow**

These recent technological advances in Monte Carlo Simulation allow for a probability power grid, which can drive asset allocation, retirement models, and valuations on everything from laptop computers to Blackberries and iPads.

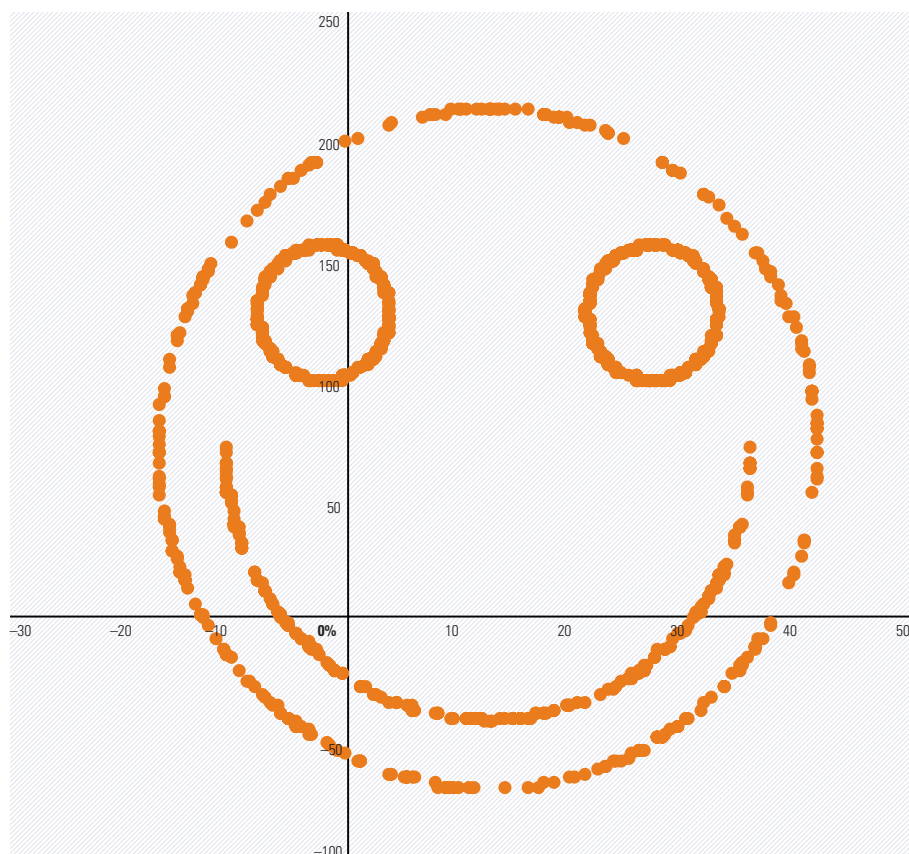
Furthermore, Monte Carlo models built with DISTs are also highly flexible, allowing for almost any type of return distribution or underlying probability model. Today, in light of the recent global financial crisis, there is much debate about how to best model the probability distributions of asset class returns.

Some researchers are proposing that we replace models based on the bell curve or normal distribution (which are tractable from a theoretical perspective), with fat tail models in which extreme events occur (which require simulation to analyze). Others argue that the models based on the normal distribution are adequate. Distribution Strings are agnostic regarding this debate.

Similarly, there is debate about the usefulness of correlation matrixes to represent the co-movements of asset class returns, with many arguing that during down markets, asset classes become more correlated. Again, the DIST approach allows any pattern of co-movements to be modeled. As an extreme, the scatter plot in Exhibit 3 of asset classes HAP and PY is a "happy face," which is certainly a type of relationship. Although the

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**Exhibit 3: Cumulative Mutual Fund Asset Class Flows Since 2007**



correlation coefficient between HAP and PY is almost zero, compressing the underlying data into a pair of DISTs preserves the relationship in its entirety.

The ability to model nonlinear relationships between return distributions has important real-life applications. Consider Exhibit 4, which is a scatter plot of returns on a stock index and a call option on the index. The DIST approach allows us to preserve the exact “hockey stick” relationship among the returns of these two assets which cannot be captured by a correlation coefficient. This is important if options are being considered as part of the portfolio.

These examples illustrate the importance of preserving underlying relationships among assets when creating a Monte Carlo model out

of DISTs. Sets of DISTs that preserve such relationships are said to be “coherent.” The creation of coherent DIST libraries is one of the most important functions of Probability Management, a field devoted to managing databases, not of numbers, but probability distributions. (See [www.ProbabilityManagement.org](http://www.ProbabilityManagement.org).)

#### Where to Find It

The power of DIST technology is beginning to appear in several programming tools for the computer-savvy investment professional. It is currently supported by three software add-ins: Risk Solver (Solver.com), and XLSim (VectorEconomics.com). The last is a multidimensional modeling tool: Analytica, (Lumina.com). For those who want ready-to-use interactive asset-allocation software with Monte Carlo models, Morningstar is in the

process of creating new tools based on the DIST technology. In the near future, it will be possible to include many types of distributions, including those that model the occasional financial crisis, in an interactive environment on the desktop or laptop. ■■■

#### References

- 1 Sam Savage, *The Flaw of Averages*, John Wiley & Sons, 2009.
- 2 Alan L Lewis., Sheen T. Kassouf, R. Dennis Brehm, and Jack Johnston, “The Ibbotson-Sinquefield Simulation Made Easy,” *Journal of Business*, vol. 53, 1980.
- 3 The Central Limit Theorem states that when a large number of statistically independent variables with the same distribution are added, the result is a normal (bell-shaped) distribution, regardless of the distribution of the underlying variables. An exception to the theorem occurs with the sort of fat tail distributions discussed in Paul D. Kaplan, “Déjà Vu All Over Again,” *Morningstar Advisor*, February/March 2009.
- 4 For a description of the model and the formulas, see pp. 113–118 in the *2010 Ibbotson Stocks, Bonds, Bills, and Inflation Classic Yearbook* published by Morningstar, Inc.
- 5 In “A Sustainable Spending Rate without Simulation,” (*Financial Analysts Journal*, November/December 2005), Moshe Milevsky and Chris Robertson present a probabilistic formula for sustainable spending rate, which they derive under a set of assumptions, one of which is that the spending rate is constant. However, for other spending patterns, such as including occasional lump-sum amounts to finance, say, a child’s wedding, a grandchild’s education, or a vacation home, in addition to regular spending, Monte Carlo Simulation remains the only practical option.
- 6 See note 4.
- 7 Michele Gambera, “It’s a Long Way to Monte Carlo,” *Business Economics*, July 2002.

Exhibit 4: Ibbotson-Sinquefield Simulation Chart

